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A SIMULATION FOR GRAVITY FINE STRUCTURE RECOVERY FROM HIGH-LOW GRAVSAT SST DATA

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16. Abstract

Covariance error analysis techniques were applied to investigate estimation strategies for the High-Low SST mission for accurate local recovery of gravitational fine structure, considering the aliasing effects of unsolved for parameters. Surface density blocks of 5° x 5° and $2\frac{1}{2}^\circ$ x $2\frac{1}{2}^\circ$ resolution were utilized to represent the high order geopotential with the drag-free GRAVSAT configured in a nearly circular polar orbit at 250 km. altitude. GEOPAUSE and geosynchronous satellites were considered as high relay spacecraft. The recovery of local sets of density blocks using independent short data arcs demonstrated that the estimation strategy of simultaneously estimating a local set of blocks covered by data and two "buffer layers" of blocks not covered by data greatly reduced aliasing errors. It is demonstrated that knowledge of gravitational fine structure can be significantly improved at 5° x 5° resolution using SST data from a High-Low configuration with reasonably accurate orbits for the Low GRAVSAT. The gravity fine structure recoverability of the High-Low SST mission is compared with the Low-Low configuration and shown to be superior.

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1.0 INTRODUCTION

Obtaining global knowledge of the earth's gravitational fine structure is necessary for understanding solid earth and ocean dynamics, and is a major objective of NASA's Earth and Ocean Dynamics Application Program [13]. Several satellite mission configurations and data types have been proposed for this task, including satellite altimetry, satellite gravity gradiometry, and satellite-to-satellite tracking. A review of these concepts for gravity field determination has been given by Argentiero and Lowrey [3]. The feasibility of utilizing a satellite-to-satellite tracking mission configured as two low altitude drag-free GRAVSAT spacecraft in identical orbits ("Low-Low") has been investigated by Estes and Lancaster [7]. This publication will be refered to as report [A]. The present study will employ the error analysis techniques of report [A] to investigate the feasibility of utilizing a "high-low" SST mission involving a high altitude relay satellite and a low altitude GRAVSAT spacecraft to recover the global gravitational fine structure.

The 10 cm accuracy requirement placed on the gravity fine structure (see [13]) necessitates an extremely large number of parameters in the field model. A simultaneous data reduction for all model parameters would be a tremendously difficult computer task, even with the most sophisticated new computer systems. This supports the idea of representing the gravitational fine structure with a local mathematical model and estimating a local area of gravitational perturbations using local data. This would, of course, require a fewer number of model parameters and make the problem more manageable. The global solution could then be built up from a set of these local solutions. This idea has been tested by simulation for SST data in report [A] and by Schwarz [15] and Hajela [8]. However, while adjusting a set of the model parameters for a local recovery, the remaining unadjusted parameters must be constrained to a priori values. Unless the gravity field parameterization and the data exhibit "orthogonality", the uncertainties of the unadjusted parameters will corrupt, or "alias", the estimates of the adjusted parameters. Orthogonality has been defined by Argentiero [2] as follows: For a given estimation procedure, the jth adjusted parameter is orthogonal to the kth unadjusted parameter if the aliasing contribution of k to j is zero. The mathematics of this concept is described in Section 4.0.

The concept of the "high-low" GRAVSAT SST mission is that the low altitude satellite is strongly perturbed by the gravitational fine structure while the high satellite is virtually unaffected, resulting in the data obtained from tracking the relative motion between them being highly sensitive to geopotential variations. Only a few geopotential terms are important at the 10 cm level for altitudes above about 5 earth radii (for non-resonance conditions).

The low altitude (250-350 km) GRAVSAT is a drag-free spacecraft constrained to follow a purely gravitational path in a polar circular orbit. The orbit geometry provides global data coverage. The high altitude satellites considered in this study are the GEOPAUSE [14] and geosynchronous.

The purpose of this investigation is to apply covariance error analysis techniques to determine under what estimation strategies the "high-low" mission is feasible for determining the short wavelength gravity field uniquely and accurately from local recovery of gravitational fine structure when the effects of aliasing and satellite state uncertainties are considered. A multi-short data arc approach to the problem is utilized (see report [A], and [4]) whereby the satellite epoch states are estimated together with the gravity parameters. For local solutions, long arcs lead to severe aliasing problems. The results obtained for the "high-low" configuration are compared with "low-low" results from report [A] in Section 6.2. To help establish the feasibility for obtaining global solutions from a composite of local solutions, a set of simulated global, long arc solutions for the "low-low" configuration are presented in Appendix A, and show compatibility with the short arc, local "low-low" results of report [A]. The drag compensation system of the GRAVSAT satellite has been assumed error free in this study, so that the results are somewhat optimistic. In a more detailed study the aliasing effect in the parameter estimation due to imperfect knowledge of the surface force compensation must be included.

2.0 SURFACE DENSITY GEOPOTENTIAL REPRESENTATION

The purpose for the present study of the high-low SST experiment is to determine the feasibility of recovering the global fine structure of the earth's gravity field as a composite of recoveries over local regions. The total geopotential is represented as

$$W = U + T$$

where the function U is assumed to be a fixed low order reference field and T represents the high order potential. We would expect in this study that dominant local gravity anomalies would be specified by a priori values in U and that the major problem of fine structure recovery would involve estimating the potential T, which would exhibit no local large amplitude anomalous behavior.

Following report [A], we choose to represent the high order geopotential by local surface density blocks specifying a fictitious surface density layer,

$$T = \iint_{\sigma} \frac{G\rho d\sigma}{d}$$

where G is the gravitational constant, ρ is the density of the mass layer, and d is the distance from the point where T is evaluated to the integration element $d\sigma$. The parameter used to describe the high order field is $\Phi \equiv G\rho$ expressed in milligals (mgal), where

$$1 \text{ mgal} = 10^{-3} \text{ gals} = 10^{-3} \text{ cm/sec}^2 \quad .$$

For order of magnitude comparisons between the surface density parameter Φ and the expression for corresponding gravity anomalies Δg in milligals, the expression

$$\Phi = \frac{\Delta g}{2\pi}$$

is adequate. A more detailed treatment of the surface layer potential is given in paper [A] and by Schwarz [15].

For simplicity, the mass layer is considered spread on the surface of a sphere with radius equal to the mean radius of the earth. The layer is modeled by individual area blocks in which the parameter Φ is constant. The disturbing potential is then

$$T = \sum_j \Phi_j \iint_{\sigma_j} \frac{1}{d_j} \, d\sigma_j$$

where the integration is over the area of the $j^{\underline{th}}$ block and the sum is over all surface blocks. The integral over each block is evaluated by dividing the block into a number of sub-blocks of area σ_{jk}, where the distance of d_{jk} is the distance from the center of the sub-block to the point of evaluation (see Figure 2.1),

$$T = \sum_j \Phi_j \sum_k \frac{\sigma_{jk}}{d_{jk}} .$$

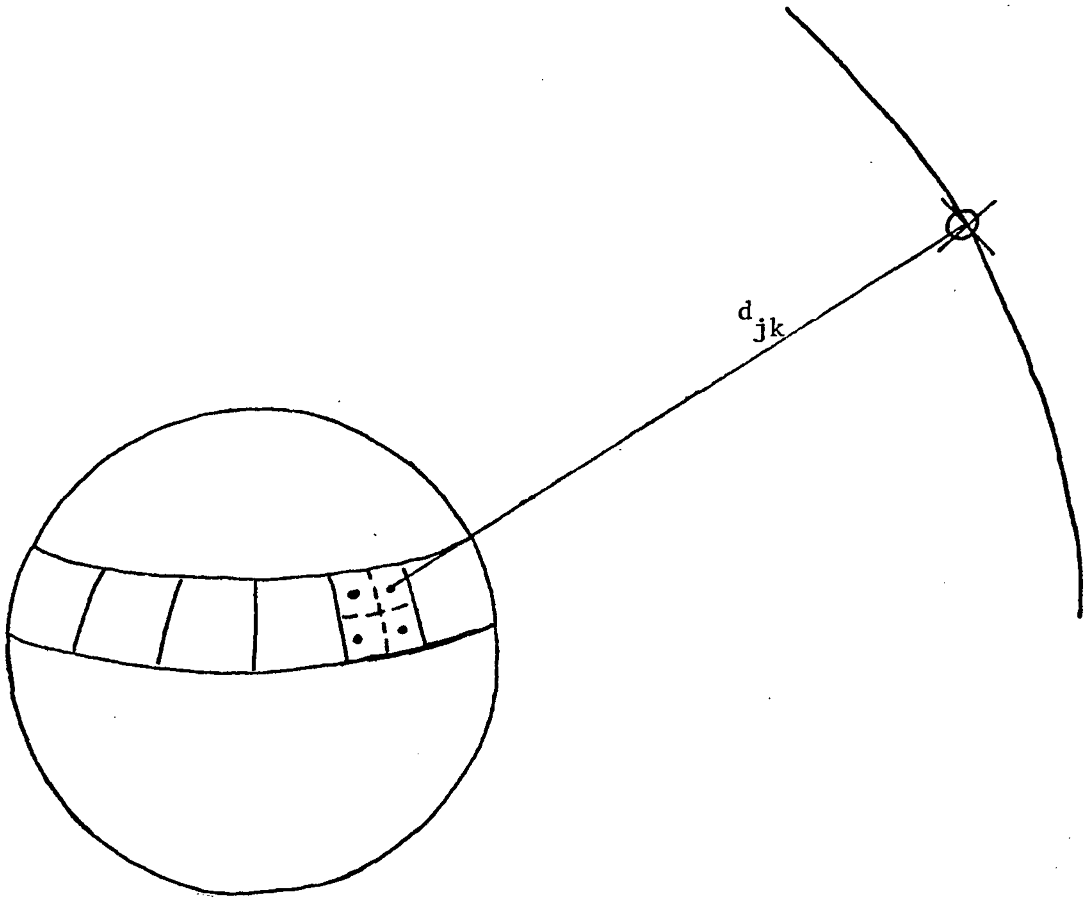

FIGURE 2.1 Subdivision of surface density blocks into sub-block approximations for numerical evaluation.

The principal portion of this study is concerned with recovery of local subsets of blocks near the equator utilizing surface blocks with boundaries defined by constant values of latitude and longitude increments (equal-angular blocks). The studies involving global recoveries and recoveries near the poles, however, utilize equal-area blocks. The algorithm implemented into the GEOMAP system for equal-area blocks is principally that due to Morrison [12].

3.0 COMPUTER SOFTWARE

The GEOMAP/ERODYN software system as developed in report [A] was the basic analysis tool utilized for this covariance error analysis study. GEOMAP is a large scale orbital parameter estimation and mission analysis program developed from the GEODYN program [5], while ERODYN is an orbital and geodetic error analysis program designed to be operated directly from GEOMAP output. To perform an error analysis of the GEOMAP estimation algorithm, ERODYN requires the estimation normal matrix output from GEOMAP. Moreover, to investigate by error analysis the partitioning of a parameter set into adjust and unadjust (alias) parameters, the total parameter set must be adjusted in a GEOMAP run so that all parameters are included in the normal matrix. This normal matrix, obtained from a single GEOMAP run, is then partitioned into appropriate adjust and unadjust sets within ERODYN. The setting of a priori parameter sigmas and the partitioning of the total parameter set into adjust and unadjust parameters within ERODYN permits a great many error analysis runs to be made from a single GEOMAP normal matrix.

The ERODYN program is designed as a single arc error analysis package. The multi-arc capability required for the short data arc recovery simulation is obtained by concatenating the normal matrices for each arc generated by GEOPMAP. For example, let A_1 and A_2 denote the portion of the normal matrix corresponding to the arc dependent parameters for arcs one and two, respectively, and let C_1 and C_2 denote that portion corresponding to parameters that are common to each arc. The two single arc normal matrixes are then

$$n_1 = \begin{bmatrix} A_1 & C_{A_1} \\ C_{A_1}^T & C_1 \end{bmatrix}$$

$$n_2 = \begin{bmatrix} A_2 & C_{A_2} \\ C_{A_2}^T & C_2 \end{bmatrix}$$

and the concatenated matrix for both arcs is

$$n = \begin{bmatrix} A_1 & 0 & C_{A_1} \\ 0 & A_2 & C_{A_2} \\ C_{A_1}^T & C_{A_2}^T & C_1 + C_2 \end{bmatrix}$$

The large dimensional multi-arc normal matrix is suitable for analysis by the linear algebra capability of ERODYN.

Some modifications to the GEOMAP/ERODYN system were implemented for the efficient performance of this study. These modifications included a fully general data simulation mode and a multi-satellite SST observability test algorithm in GEOMAP and a dynamic partitioning scheme for the normal matrix inversion operation in ERODYN to reduce the large computer core requirement for the large parameter sets involved in this study. In addition, the single arc normal matrix concatenation program was modified to accomodate multi-satellite/ multi-arc configurations. Independent verification of the program modification was aided by comparing test runs using this software with results obtained from an operating BTS program designed to analyze state vector observability obtained from simulated SST data [6].

4.0 COVARIANCE ERROR ANALYSIS

The error analysis for the Bayesian least squares estimator in GEOMAP is described in the ERODYN Program Mathematical Description [9] and in report [A]. However, for completeness we reproduce the main points of the mathematical treatment.

Let y be an n dimensional vector of observations modeled by the nonlinear equation

$$y = F(X,S) + \varepsilon$$

where X and S are parameters of the dynamical system. Here X denotes the parameter set to be adjusted (estimated) and S denotes the parameter set which is unadjusted, or constrained to constant values in the estimation process but whose uncertainties are to be considered in computing the covariance matrix of the estimate. The vector ε represents the observation noise, which is mean zero and statistically independent. Linearizing about a nominal solution of the parameters X_N and S_N yields

$$y = C \begin{bmatrix} \delta X \\ \\ \delta S \end{bmatrix} + \varepsilon$$

$$= A\delta X + B\delta S + \varepsilon$$

where

$$\delta y = y - F(X_N,S_N)$$

$$\delta X = X - X_N$$

$$\delta S = S - S_N$$

$$A = \frac{\partial F(X,S)}{\partial X}\Bigg|_{X_N,S_N}$$

$$B = \left. \frac{\partial F(X,S)}{\partial S} \right|_{X_N, S_N}$$

and

$$C = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$$

Here X and S denotes the actual values of the adjusted and unadjusted parameters and $\delta X_a = X - X_a$ represents the difference between the actual and a priori values of the adjusted parameters.

The normal matrix for all parameters is given by

$$n = C^T W C$$

$$= \begin{bmatrix} A^T WA & A^T WB \\ B^T WA & B^T WB \end{bmatrix}$$

where the weight matrix W is the inverse of the observation noise covariance matrix and is assumed to be diagonal, i.e.

$$W^{-1} = E(\varepsilon\varepsilon^T) = \begin{bmatrix} \sigma_1^2 & & 0 \\ & \ddots & \\ 0 & & \sigma_n^2 \end{bmatrix}$$

where E denotes expected values and σ_i is the ith observation noise sigma.

The Bayesian least squares estimate of X is given by

$$\hat{\delta X} = (A^T WA + P_a^{-1})^{-1} A^T W \delta y + (A^T WA + P_a^{-1})^{-1} P_a^{-1} \hat{\delta X}_a$$

where

$$\hat{\delta X}_a = X_a - X_N$$

and

$$P_a = E[(X_a - X)(X_a - X)^T]$$

is the covariance matrix of the a priori estimate X_a. Substitutions yield the error

$$\Delta X = \hat{\delta X} - \delta X = (A^T WA + P_a^{-1})^{-1}(A^T WB\delta S + A^T W\varepsilon - P_a^{-1}\delta X_a)$$

showing the three distinct components due to aliasing, measurement noise, and a priori uncertainties.

Under the assumption that δS, ε, and δX_a are uncorrelated errors, the covariance matrix of the adjusted parameters is given by

$$P \equiv E[(\hat{X} - X)(\hat{X} - X)^T] = (A^T WA + P_a^{-1})^{-1}$$

$$+ (A^T WA + P_a^{-1})^{-1}(A^T WB)\ V_S\ (B^T WA)(A^T WA + P_a^{-1})^{-1}$$

where V_S is the diagonal covariance matrix of the unadjusted parameters,

$$V_S = E(\delta S \delta S^T) \quad .$$

The sensitivity matrix of adjusted to unadjusted parameters is

$$\mathcal{S} = \frac{\partial \Delta X}{\partial \delta S} = (A^T WA + P_a^{-1})^{-1} A^T WB$$

so that the aliasing error to the ith adjusted parameter due to the jth unadjusted parameter is

$$\sigma_{ij} = \mathcal{S}_{ij} \quad \sigma_{S_j} \quad .$$

The ith diagonal element of P, or the variance of the ith adjusted parameter is then given by

$$\sigma_i^2 = \left[(A^T WA + P_a^{-1})^{-1}_{ii}\right] + \sum_j (S_{ij}\sigma_{S_j})^2$$

so that the standard deviation of the ith adjusted parameter is the root sum square (RSS) of the standard deviations due to data noise and a priori uncertainties and the standard deviations due to each unadjusted parameter.

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TABLE 4.1

Geopause:	r = 29431200. m
	h = 23,053.045 km.
	e = 0
	i = 90°
Geostationary:	r = 42164541.3 m
	h = 35,786.386 km.
	e = .000388
	i = 1.01°
GRAVSAT:	r = 6628155.0 m
	h = 250 km.
	e = 0
	i = 90°

The data rate assumed in this error analysis study for local recovery is one observation every five seconds along the satellite path and four satellite passes per surface density block for all cases using the 5° x 5° set of equal-angular blocks centered on the equator. Short arc passes are ascending and decending. The spacing of satellite passes is slightly different for the 2 1/2° x 2 1/2° density block and the 5° x 5° equal-area density block studies and will be fully described in Section 6.0. Each data arc is treated as being independent. The GEOMAP program (report [A]) generates each arc normal matrix separately and then for a given set of short arcs the arc normal matrices are concatenated into a single normal matrix for error analysis purposes.

The SST range-rate signal signature for short arc passes of the "low-low" GRAVSAT, the GRAVSAT/GEOPAUSE and the GRAVSAT/GEOSTATIONARY as the GRAVSAT passes over a 5° x 5°, 1 mgal surface density block located on the equator is shown in Figures (4.1-4.6). The marker (▲) along the time axis denotes the time that the low satellite is directly over the center of the density block. In these figures all passes are ascending arcs with the low satellite 15° below the equator at epoch. The angle in parenthesis designated ΔM in the legend on Figures 4.2-4.5 represents the mean anomaly difference of the high GEOPAUSE and low GRAVSAT at the epoch, in degrees. Thus, (-40°) indicates that at epoch the low satellite is 40° ahead and moving away from the high satellite.

The signal strength is seen to be greater for the "high-low" configuration than the "low-low" configuration. This will be displayed in the error analysis cases of Section 6.0. Moreover, the magnitude and shape of the "high-low" signal varies with the geometry of the high and low orbit at epoch. The data arcs generated for the error analysis studies of Section 6.0 were designed to be a realistic mix of geometries for the high and low orbits. This will be discussed more fully in Section 6.0.

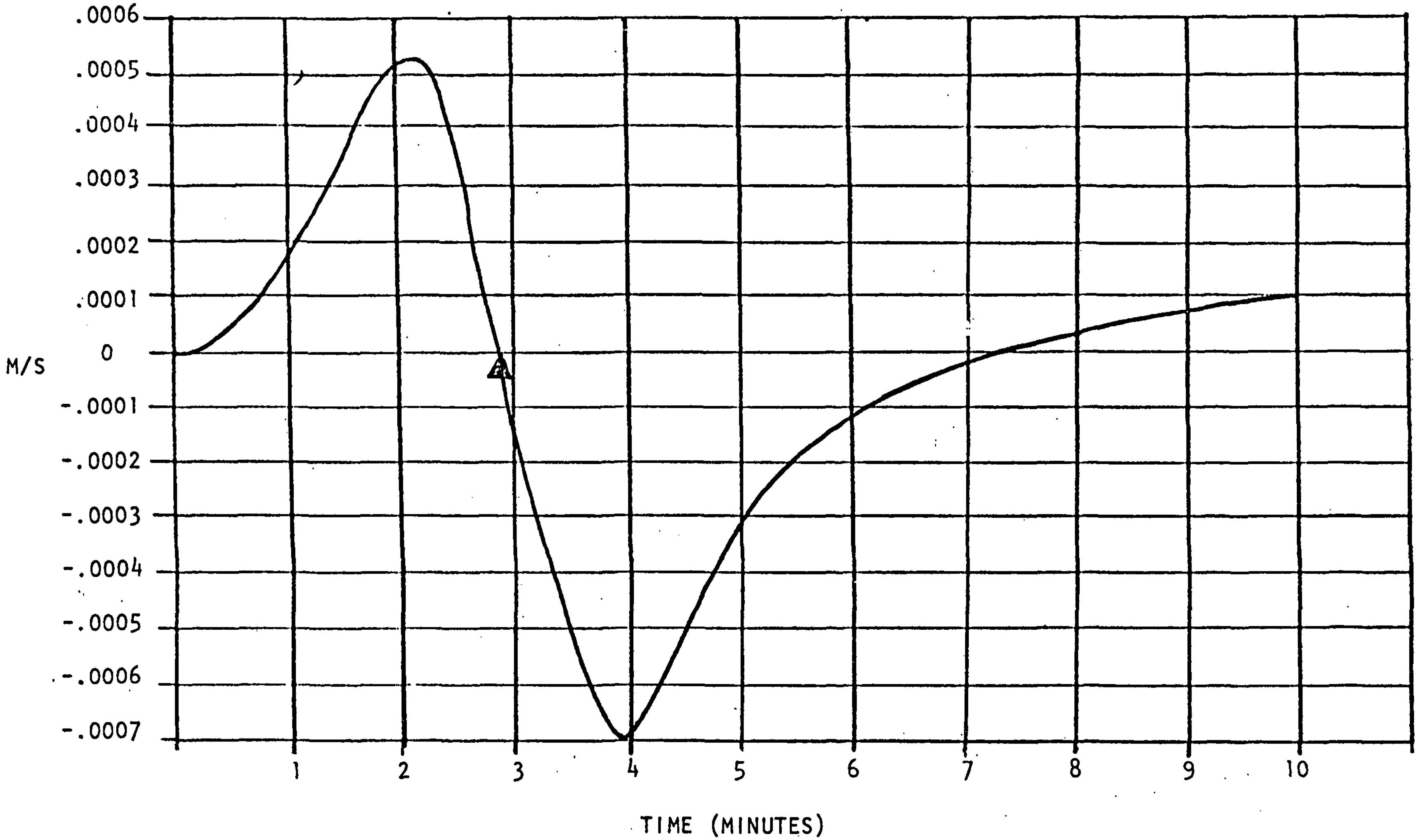


FIGURE 4.1 "Low-Low" GRAVSAT SST Signal Over A 1 MGAL 5° x 5° Density Block.

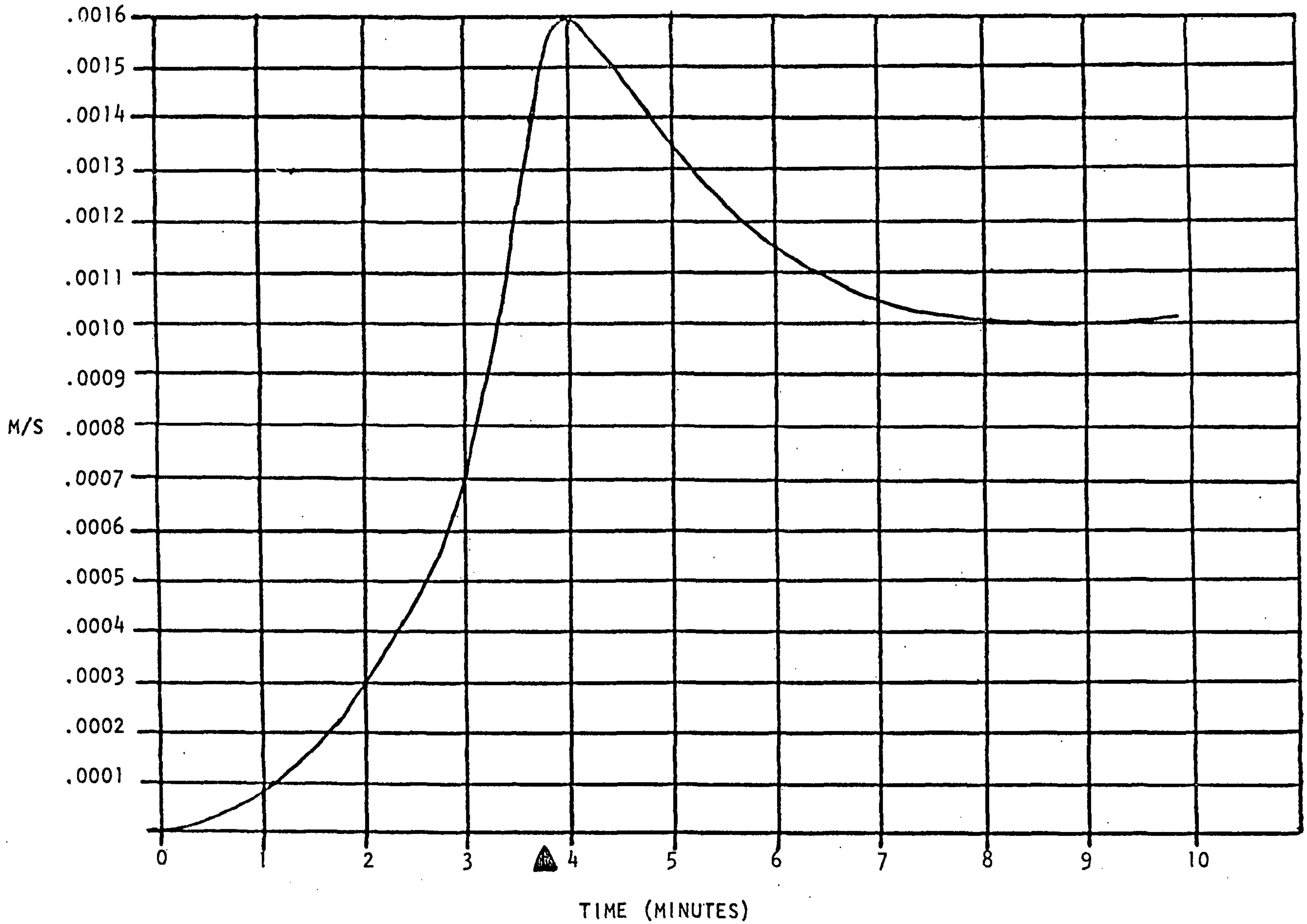


FIGURE 4.2 GRAVSAT/GEOPAUSE SST Signal Over A 1 MGAL 5° x 5° Density Block [Coplanar, ΔM =(-40°)].

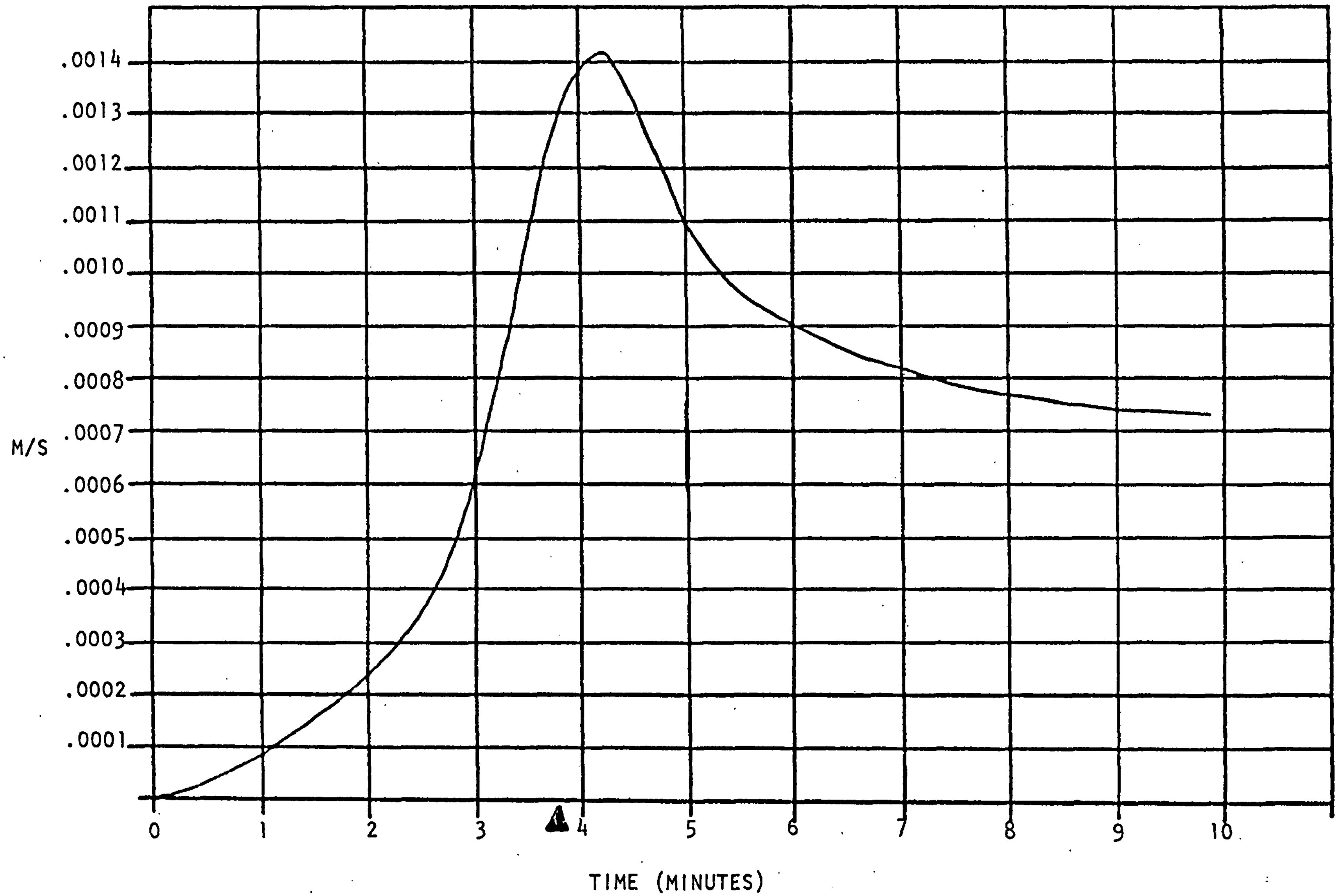


FIGURE 4.3 GRAVSAT/GEOPAUSE SST Signal Over A 1 MGAL 5° x 5° Density Block [Planes 30° Offset, ΔM =(-40)].

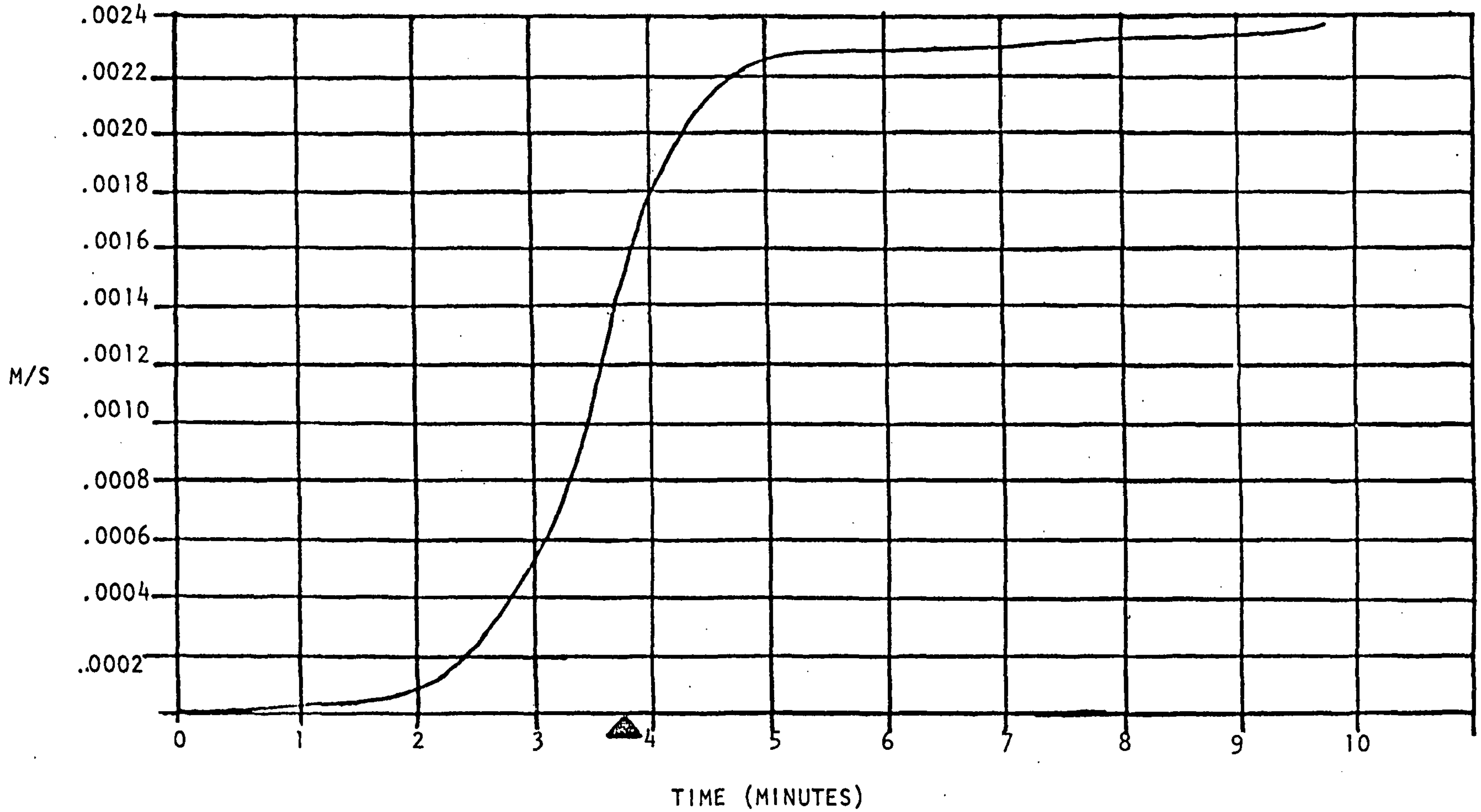


FIGURE 4.4 GRAVSAT/GEOPAUSE SST Signal Over A 1 MGAL 5° x 5° Density Block [Coplanar, ΔM = (0°)].

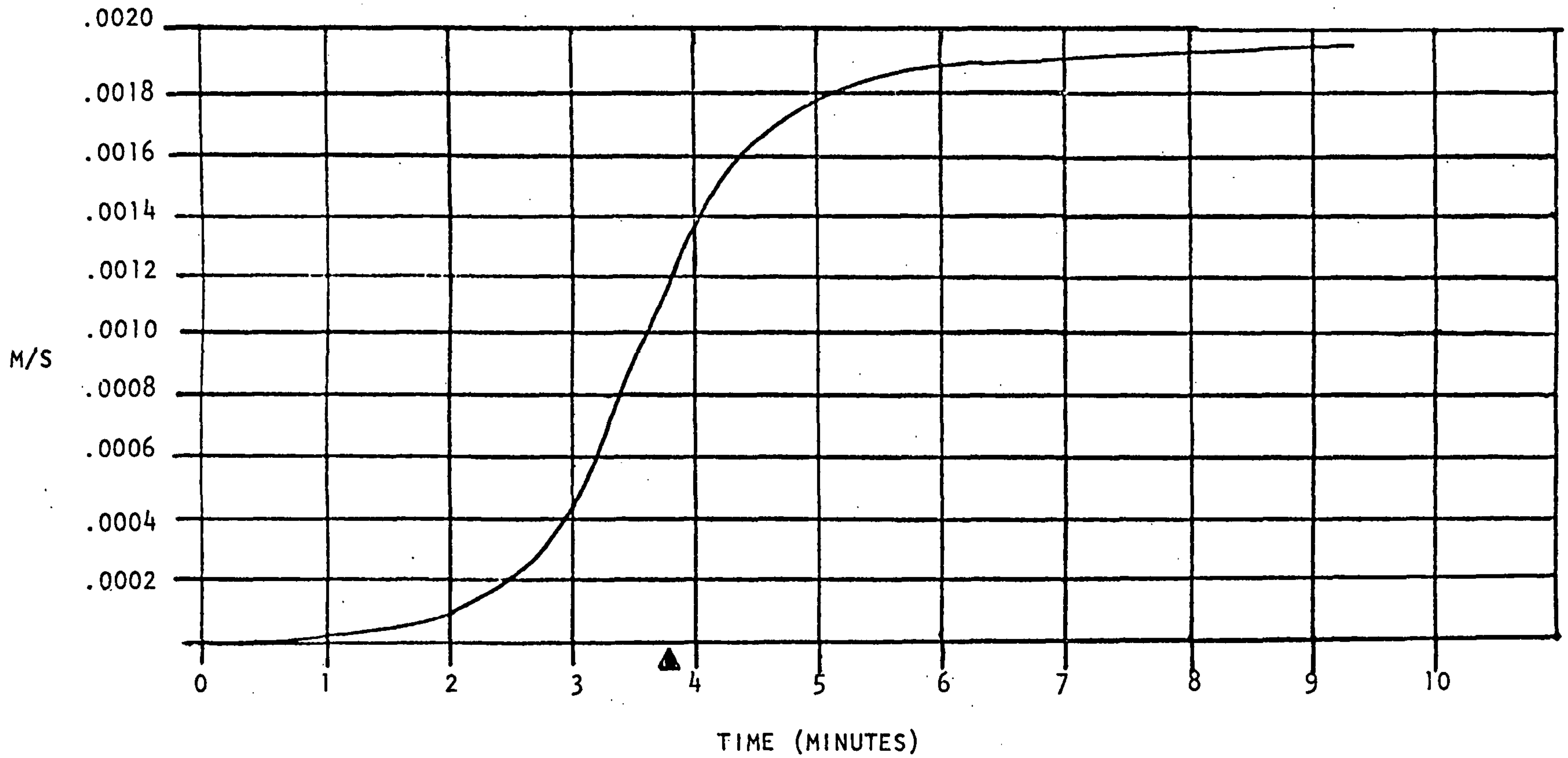


FIGURE 4.5 GRAVSAT/GEOPAUSE SST Signal Over A 1 MGAL 5° x 5° Density Block [Planes 30° Offset, ΔM = (0°)].

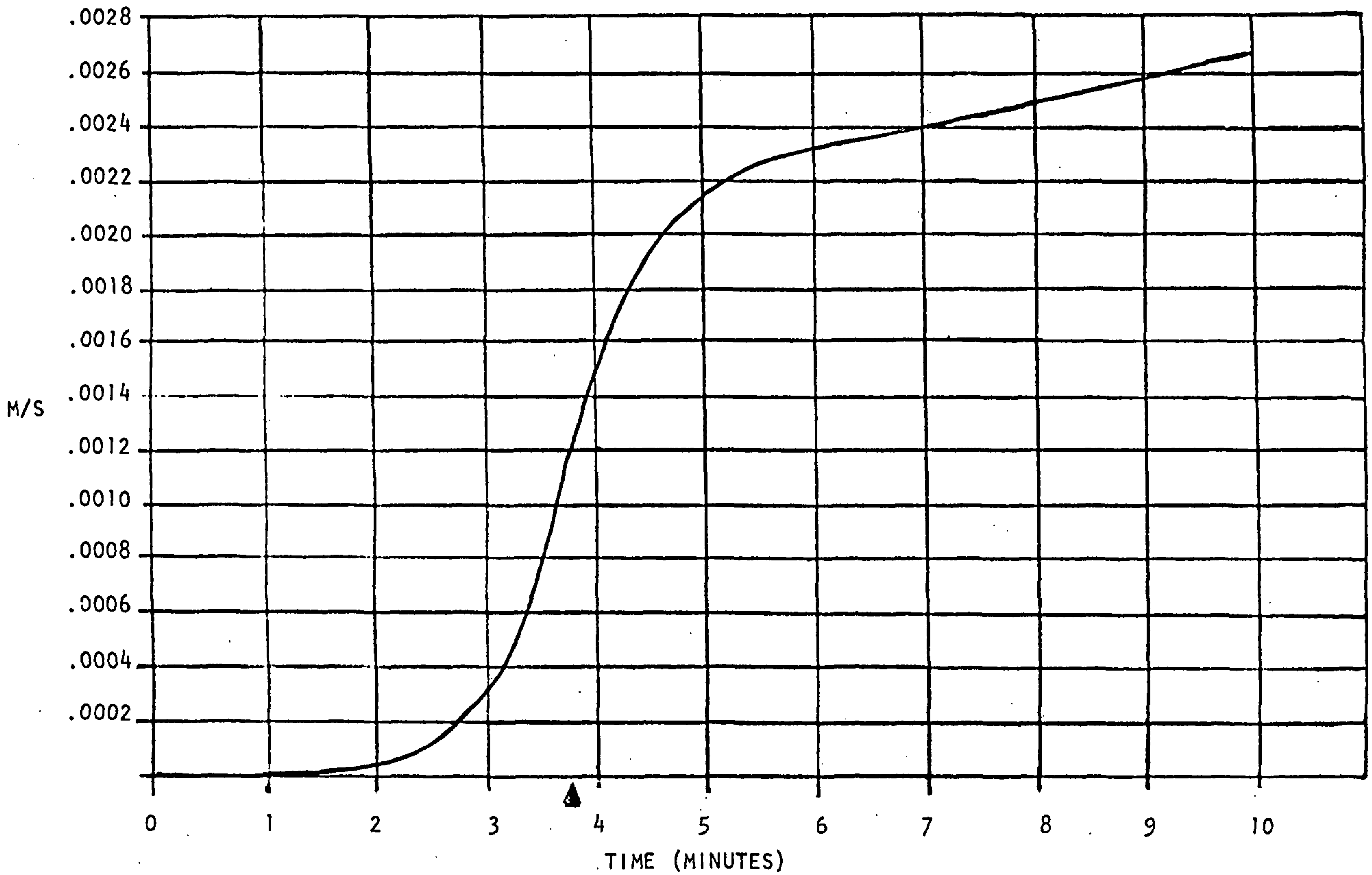


FIGURE 4.6 GRAVSAT/GEOSTATIONARY SST Signal Over A 1 MGAL 5° x 5° Density Block [High Satellite 12° East of Density Block].

6.0 DATA SIMULATION AND ERROR ANALYSIS

As demonstrated by Hajela [8] and in report [A], if a local set of density blocks is to be estimated by SST data the blocks near the edges will be badly corrupted (aliased) by nearby unadjusted blocks, whereas blocks near the center of the set may be sufficiently removed from the unadjusted blocks that their estimates may be acceptable. Thus, to estimate a given set of blocks it will be necessary to simultaneously estimate the set of interest plus a surrounding set of "buffer" blocks, whose estimates will be discarded due to aliasing. To determine the relationships between estimation accuracy and the estimation region, data region, data noise, a priori uncertainties in adjusted and unadjusted density blocks and a priori uncertainties in the satellite states we follow the approach of report [A] and utilize the method of covariance error analysis outlined in Section 4.0. It is recalled that in a covariance analysis only the covariance matrix for the estimator is calculated - the least squares estimation is postulated and not actually performed. Parameters in the "solve for" or adjust mode are assumed to be estimated in the postulated least squares adjustment, while the parameters in the "consider" or unadjust mode are assumed to affect the functionality between the adjusted parameters and the observations but are not estimated.

The error analysis results are organized into six sets:

SET I: The "high-low" GRAVSAT/GEOPAUSE configuration simulated with twenty-one short data arcs over 225 5° x 5° equal angular blocks arranged in a 15 block by 15 block square centered on the equator with data coverage over the center 25 blocks.

SET II: The "high-low" GRAVSAT/GEOSTATIONARY configuration simulated with twenty-one short data arcs over the same density block configuration as SET I. The GEOSTATIONARY satellite is positioned at the center of the local set of blocks.

SET III: The "high-high-low" GRAVSAT/GEOSTATIONARY configuration simulated with twenty-one short data arcs over the same density block configuration as SET I. The two GEOSTATIONARY satellites are positioned 60° to the east and west of the center of the local set of blocks and take independent, simultaneous SST range-rate measurements of the GRAVSAT.

SET IV: The "high-low" GRAVSAT/GEOSTATIONARY configuration simulated with ten short data arcs over 225 2 1/2° x 2 1/2° equal angular blocks arranged in a 15 block by 15 block square centered on the equator with data coverage over the center 25 blocks.

SET V: The "high-high-low" GRAVSAT/GEOSTATIONARY configuration simulated with twelve short data arcs over 185 5° x 5° equal-area blocks arranged in 8 latitude bands centered on the north pole with data coverage over the first three bands about the pole. The two GEOSTATIONARY satellites are positioned 180° apart.

SET IV: The "low-low" GRAVSAT configuration simulated with twenty-one short data arcs over the same density block configuration as SET I.

The total parameter set for the error analysis consists of six state parameters for each satellite for each arc plus the entire set of surface density parameters. The sensitivity matrix C, consisting of the partial derivatives of the observations with respect to the parameter set, is evaluated along a nominal trajectory where the a priori values of all surface densities have been set to zero. The resulting normal matrix is adequate for linear error analysis purposes and provides a substantial savings of computer time.

6.1 DATA COVERAGE

The simulated satellite-to-satellite range-rate data for the six sets of error analysis studies described in 6.0 all were generated from a two body plus J_2 force model. Using the GEOMAP program with an eleventh order Cowell predictor-corrector integrator and a 20 second fixed step size to integrate the equations of motion. The data, generated at 5 second intervals, was then contaminated with random noise of standard deviation .05 cm/sec. The 5 second data interval produces approximately 15 observations per equatorial 5° x 5° equal-angular denisty block per arc. However, the high satellite geometry, the data arc spacing, and the geographic region covered by data varied among the six sets. The particular features of the data arcs for each of the sets will be described in this subsection. It is to be noted that the simulation is idealized in that there are no transmission delays at the satellites, all measurements are instantaneous,and the drag compensation system of the GRAVSAT has been assumed error free. This means that error analysis results will be somewhat optimistic, but will serve to determine feasibility.

SET I:

The data coverage for the coplanar, circular, polar orbiting GRAVSAT/GEOPAUSE configuration consists of 21 short arc data passes of the low satellite over the center 25 blocks of the 225 5° x 5° equal-angular blocks centered on the equator. The passes are alternately ascending and decending with approximately 1.2° spacing, giving approximately four data passes per block. The ground track of the low GRAVSAT over the data region is displayed in Figure 6.1. The epoch for each data arc is at -15° latitude for ascending and +15° latitude for descending passes, while the data region extends between -12 1/2° and +12 1/2° latitude. Because the period of the GRAVSAT at 250 km altitude is approximately 90 minutes and the period of the GEOPAUSE is approximately 14 hours, there is a highly variable geometry possible between the high and low satellites. Using a line of sight observability test between the satellites, a realistic mix of configurations was calculated which would provide the desired data coverage. The orientation of the high satellite relative to the low satellite for the 21 passes is described in Table 6.1. As defined in Section 5.0, ΔM represents the mean anomaly difference of the high GEOPAUSE and low GRAVSAT at the epoch, in degrees. A positive ΔM indicates that the low satellite is "moving toward" the high satellite, while a negative ΔM indicates that the low satellite is "moving away".

Two sets of 21 data arcs identical to the above coplanar configuration with the exception that the orbit plane of the GEOPAUSE was offset from the plane of the GRAVSAT by 30° and 45°, respectively, were generated to study the effects of adding out-of-plane components in the data.

Table 6.1 Orientation of the 21 Data Arcs for the GRAVSAT/GEOPAUSE Configuration

\+ means Low satellite moving toward High satellite

\- means Low satellite moving away from High satellite

Arc	A - Ascending D - Descending	ΔM (at epoch) in degrees
1	A	-40
2	D	-50
3	A	+60
4	D	+55
5	A	+90
6	D	+16
7	A	-40
8	D	+ 4
9	A	+50
10	D	-40
11	A	+10
12	D	+55
13	A	- 5
14	D	+ 4
15	A	+90
16	D	+16
17	A	+50
18	D	+55
19	A	+10
20	D	+40
21	A	+90

SET II:

The data coverage for the GRAVSAT/GEOSTATIONARY "high-low" configuration consists of 21 short arc data passes of the low satellite over the center 25 blocks of the 225 5° x 5° equal-angular blocks centered on the equator. The epochs and ground tracks of the low satellite are identical to SET I, described above and displayed in Figure 6.1. The high geostationary satellite is located at the center of the 25 block data region.

SET III:

The data coverage for the "high-high-low" GRAVSAT/GEOSTATIONARY configuration is identical to that of SET II with the exception that there are two simulated geostationary satellites located 60° to the east and west of the center of the 25 blocks covered by data, respectively. All 21 passes of the GRAVSAT are simultaneously observable by both high tracking satellites, and SST range-rate observations to the low satellite are generated independently.

SET IV:

The data coverage for the GRAVSAT/GEOSTATIONARY "high-low" configuration consists of 10 short arc data passes of the low satellite over the center 25 blocks of 225 2 1/2° x 2 1/2° equal-angular blocks centered on the equator. The passes are alternately ascending and descending with approximately 1.2° spacing, giving approximately two data passes per block. The ground track of the low GRAVSAT over the data region is displayed in Figure 6.2. The epoch for each data arc is at -15° latitude for ascending and + 15° latitude for descending passes, while the data region extends between -6 1/4° and +6 1/4° latitude. The high geostationary satellite is located at the center of the 25 block data region.

FIGURE 6.1 Ground Track of 21 GRAVSAT Data Arcs Over 25 5° x 5° Equal-Angular Density Blocks.

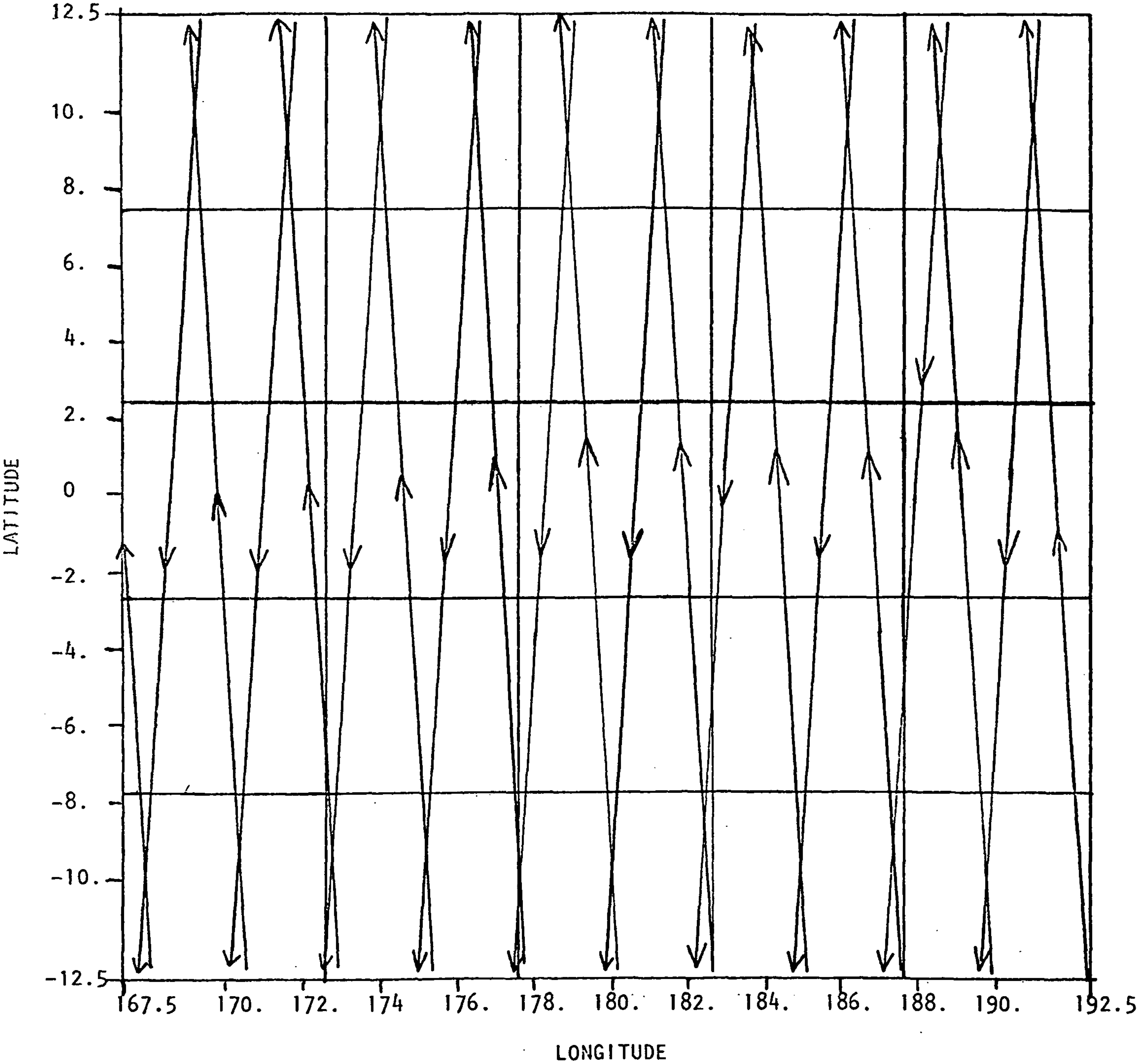
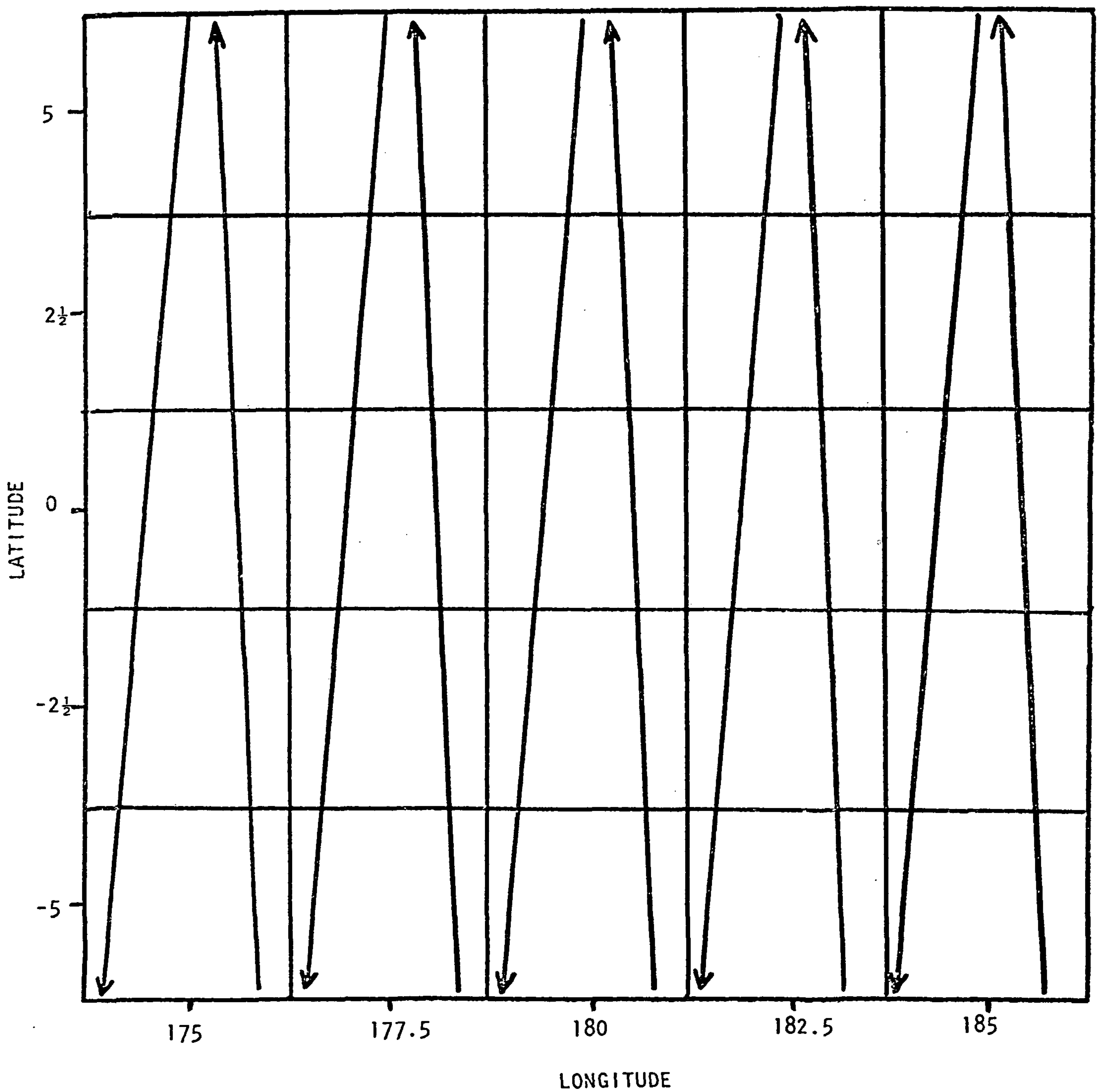


FIGURE 6.2 Ground Track of 10 GRAVSAT Data Arcs Over 25 2½° x 2½° Equal Angular Density Blocks.



SET V:

The data coverage for the GRAVSAT/GEOSTATIONARY "high-high-low" configuration consists of 12 short arc data passes of the low satellite over the first three latitude bands of 5° x 5° equal-area blocks centered on the north pole. The passes are alternately ascending and descending with approximately 15° spacing, giving 2 data passes per block in the second latitude band. The ground track of the low GRAVSAT over the data region is displayed in Figure 6.3. The epoch for each arc is at 70° latitude, while the data region extends from 75° latitude to the pole. The two relay satellites are located 180° apart at 90° and 270° longitude, respectively. The data was simulated such that the low satellite was observable only to the relay satellite at 90° longitude when its longitude was between 0° and 180°, and to the relay satellite at 270° longitude when its longitude was between 180° and 360°. Note that the polar cap receives approximately 6 times more data than the blocks in the second latitude band (See Figure 6.3), while the second latitude band receives approximately 2 times more data than the blocks in the third latitude band.

SET VI:

The data coverage for the GRAVSAT "low-low" configuration (see report [A]) consists of 21 short arc data passes of the two low satellites, separated by a geometric angle of 6° at 250 km. altitude, over the center 25 blocks of the 225 5° x 5° equal-angular blocks centered on the equator. All data was generated on ascending passes with approximately 1.2° spacing, giving approximately four passes per block. The epoch for each data arc is at -15° latitude, while the data region extends between -12 1/2° and +12 1/2° in latitude. The data passes are symmetrically defined by the ground track of the midpoint between the two satellites.

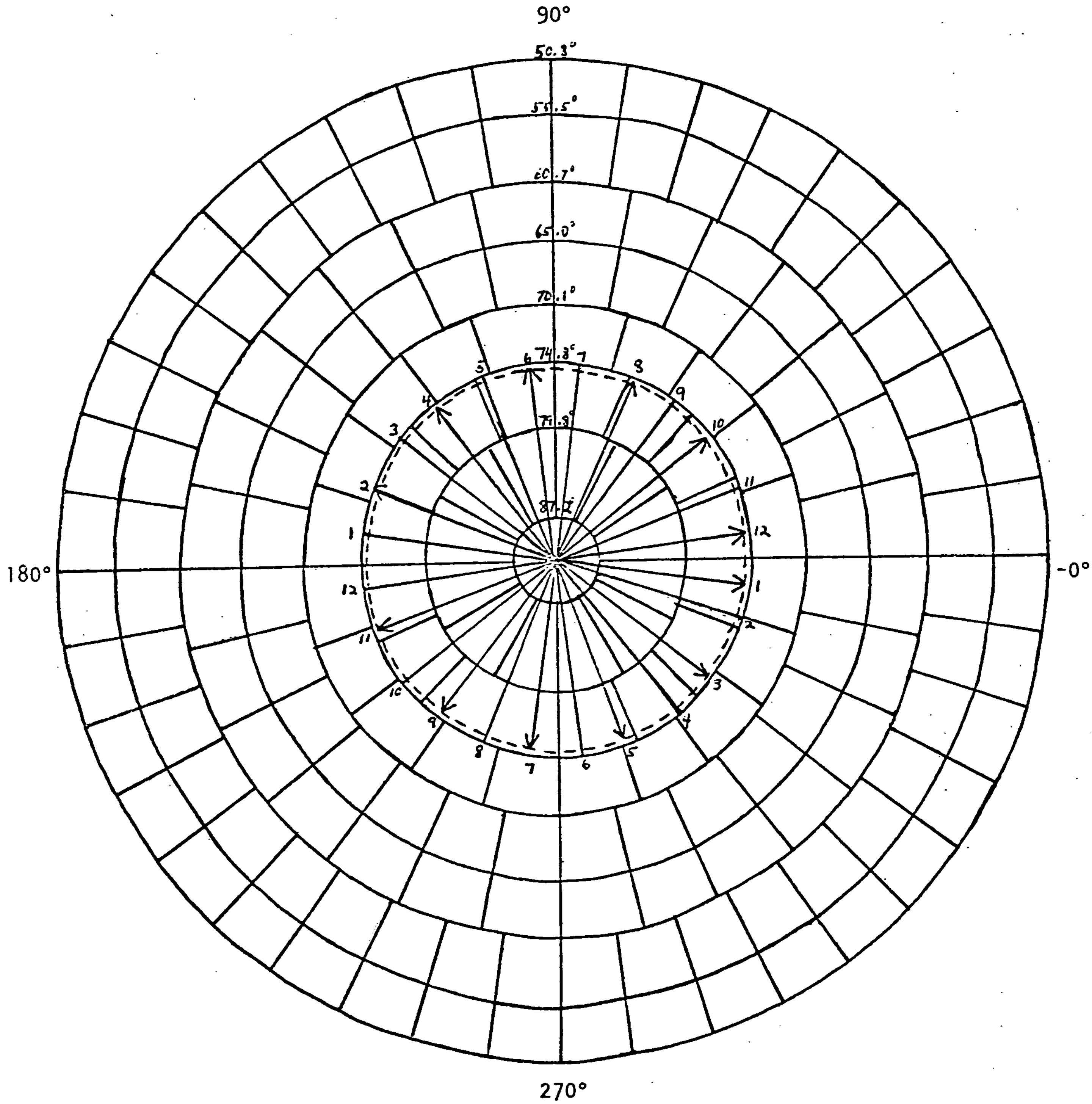


FIGURE 6.3 Ground Track of 12 Data Arcs For GRAVSAT - 2 GEOSTATIONARY Polar Configuration of 5° Equal-Area Blocks. The Geostationary Satellites Are Located at 90° and 270° Longitude.

6.2 ERROR ANALYSIS STUDY

The error analysis results were generated under the assumption that a priori satellite epoch state errors reflect the orbital knowledge obtained from other tracking means. No ground tracking data was included in the study. Assumptions concerning the SST range-rate data and the geometry of the satellite configurations were discussed in Section 6.0 and 6.1. Any assumptions that are particular to any of the six error analysis sets will be described individually in the discussion of the error analysis results of those sets.

In the tables of solution sets I-VI displaying error sigmas, the dashed perimeter encloses the data region, while the set of blocks containing two numbers are the adjusted density blocks. The two numbers within each adjusted block represent the total RSS error sigma for that block and the error sigma for that block due to aliasing by unadjusted blocks in milligals (mgal), i.e.

$$\sigma_{TOTAL} \; / \; \sigma_{ALIAS} \; .$$

The error sigma due to data noise and a priori parameter uncertainties is then

$$\sigma_{NOISE} = \sqrt{\sigma^2_{TOTAL} - \sigma^2_{ALIAS}} \quad .$$

The numbers in unadjusted blocks represent the aliasing contribution of that block to the center adjusted block. The signs on these aliasing contributions reflect the influence of the sensitivity matrix $S = \frac{\partial \Delta X}{\partial \Delta S}$. In the tables of solution sets I-VI displaying correlations, the number within each adjusted block is the correlation of that adjusted block with the center adjusted block.

The results presented in Set VI for the "low-low" configuration are extensions of results obtained in report [A] and are included here for purposes of comparison with the "high-low" mission. As in the case of the "low-low" configuration, it was found necessary to simultaneously solve for the epoch state vector of the low satellite along with the density blocks to obtain a reasonable recovery for the blocks. The epoch state errors of the high satellite produced a much smaller effect on the recovery, however, and studies in Sets I and II demonstrate the result of treating the epoch state vector of the high satellite as unadjusted. In solution Sets I-VI, however, the high satellite state vector will be included with the low satellite state vector in the adjust mode unless explicitly stated otherwise.

SET I:

Solution Set I treats the "high-low" GRAVSAT/GEOPAUSE configuration as described in Section 6.1. As in report [A], the center block is the main focus of attention in these comparative studies for quantitative aliasing contributions. This error analysis set will investigate the effectiveness of reducing aliasing errors by adding "buffer layers", the effects of high satellite state errors on the solution, and the relation that a priori uncertainty in the adjusted and unadjusted density blocks has on the accuracy of the estimation. The coplanar, 30° and 45° planar offset GRAVSAT/GEOPAUSE configurations are presented in Tables I.I-I.XV, Tables I.XVI-I.XIX, and Tables I.XX-I.XXII, respectively.

Table I.I displays what should be the most favorable solution for the coplanar configuration. The data region is buffered with two adjusted layers to reduce aliasing effects, the 4 mgal a priori uncertainty on both the adjusted and unadjusted blocks represents a reasonable value for 5° x 5 ° density blocks, and the satellite state for both the high and low spacecraft are assumed to be virtually error free (a priori uncertainties of both position and velocity set to the small value .001 for convenience). The resulting aliasing error of the center block is small, and the aliasing contribution of unadjusted blocks decreases rapidly with distance from the adjusted set. The dominant error source is the "noise plus a priori" contribution arising from

$$\left(A^T WA + P_A^{-1}\right)^{-1} .$$

The reduction of the aliasing effect due to unadjusted blocks by estimating "buffer layers" is demonstrated in Tables I.II-I.III. Here the satellite states are again assumed virtually error free, but the a priori uncertainty on both the adjusted and unadjusted blocks is set to the large value of 50 mgal. This greatly increases the aliasing of the adjusted set so that the effective strategy of reducing aliasing errors in the central region of interest by estimating "buffer layers" not covered by data can be clearly seen. The error of 4.55 mgal for the center block of Table I.II is almost entirely due to the aliasing contribution, as is the .76 mgal of Table I.III when a single layer is introduced. However, the double "buffer layer" of Table I.IV results in .33 mgal error composed of nearly equal "aliasing" and "noise plus a priori" components. For realistic values of unadjusted block a priori, as in Table I.I, the "noise plus a priori" component will dominate the total error, showing the two "buffer layers" for the 5° x 5° density blocks to be an effective strategy to eliminate aliasing errors in the local estimation process. It is to be noted that the "high-low" configuration produces a less sensitive range-rate data type to this aliasing problem than the "low-low" concept investigated in report [A]. In that case two "buffer layers" were required also, (compare Tables I.I and VI.I) but the addition of a single layer for the "high-low" produced a much better solution with respect to aliasing than did the addition of a single layer for the "low-low".

Tables I.V-I.XI display the recovery errors to be expected utilizing realistic values for the uncertainties of density blocks (4 mgal) and the epoch states for the high and low spacecraft. Table I.V uses state uncertainty values obtained by Koch [11] for the coplanar GRAVSAT/GEOPAUSE mission and finds recovery to an accuracy of .2 mgal. To determine the character of the recovery as a function of epoch state a priori errors, Tables I.VI-I.XI utilize multiples of the set (2m, 5m, 10m) for the position uncertainties in radial, cross track and along track (H,C,L) components for the GRAVSAT and the sets (.4m, 1m, 2m) and (20m, 50m, 100m) for the position uncertainties in HCL coordinates for the GEOPAUSE. The velocity uncertainties are calculated from the relations

$$\sigma_{v_r} = \frac{v}{r}\ \sigma_L$$

$$\sigma_{v_c} = \frac{v}{r}\ \sigma_c$$

$$\sigma_{v_L} = \frac{v}{r}\ \sigma_r$$

where $(\sigma_r, \sigma_c, \sigma_L)$ and $(\sigma_{v_r}, \sigma_{v_c}, \sigma_{v_L})$ are the position uncertainites and velocity uncertainties, respectively, in HCL coordinates, and r and v are the corresponding magnitudes of the orbital position and velocity vectors.

Comparing Tables I.IX and I.VI shows that increasing the high satellite epoch state uncertainty from (.4m, 2m, 1m) to (20m, 100m, 50m) in HCL coordinates increases the total error of the center block from .30 mgal to .33 mgal, while the aliasing component remains the same. This demonstrates the relative insensitivity of the estimation accuracy to the uncertainty of the high satellite orbit (at the 10's of meters level). Table I.XII treats the high satellite epoch state parameters of Table I.IX as unadjusted parameters resulting in a total error increase to .39 mgal, while the aliasing error component increases from .04 mgal to .26 mgal. The aliasing due to the high satellite orbital uncertainty is approximately equal to the "noise plus a priori" contribution.

Tables I.VI (or I.IX) shows that if the low satellite is assumed accurate to (2m, 5m, 10m) in HCL coordinates and the high satellite accurate to (.4m, 1m, 2m), or (20m, 50m, 100m) then a priori density block errors of 4 mgal can be reduced to approximately .3 mgal. If these orbital errors are multiplied by a factor of ten, then (Tables I.VII and I.X.) a priori density block errors of 4 mgals can be reduced to approximately 1.2 mgal.

The correlations of the adjusted blocks with the center adjusted block for the error analysis studies of Tables I.I, I.IV and I.VI are presented in Tables I.XII-I.XV, respectively. These correlations display the resolvability of 5° x 5° blocks with the "high-low" configuration for the GRAVSAT at 250 km. altitude.

The offset planes of the GRAVSAT and GEOPAUSE alters the geometry such that cross track information is also contained in the SST data. Comparing Tables I.XVI and I.XIX for a 30° offset with the corresponding coplanar solution of Table I.II and I.VI demonstrates that the additional geometry improves the recovery when the a priori uncertainty of the density blocks is large (50 mgal) and has virtually no impact when the a priori uncertainties are smaller (4 mgal). For example, the error analysis study of Table I.XVII was repeated for the coplanar configuration and the total error of the center block was found to be .71 mgal, with an aliasing component of .55 mgal. These results can be compared with the corresponding GRAVSAT/GEOSTATIONARY case of Table II.VIII which falls between the coplanar and 30° planar offset configurations.

Tables I.XVII-I.XIX demonstrate the effect of changing the magnitude of the a priori uncertainty for the unadjusted blocks while keeping the large a priori value of 50 mgal for the adjusted blocks, while Tables I.XIX-I.XXI compare the properties of "buffer layers" for the 45° planar offset recovery with the coplanar case shown in Tables I.II-I.IV.

							-.001							
							-.0008							
							.0003							
			3.98 / .88	3.91 / 1.04	3.74 / 1.24	3.77 / 1.54	3.67 / 1.54	3.76 / 1.55	3.68 / 1.24	3.84 / .96	3.96 / .74			
			3.92 / .97	3.38 / .57	1.48 / .35	1.09 / .16	1.11 / .21	1.07 / .16	1.42 / .25	2.88 / .58	3.88 / .71			
			3.85 / 1.09	1.77 / .26	4.08 / .06	.26 / .04	.26 / .04	.27 / .04	.42 / .07	1.62 / .20	3.82 / .74			
			3.78 / 1.04	1.34 / .14	.29 / .06	.18 / .05	.18 / .04	.19 / .04	.31 / .06	1.20 / .16	3.79 / .70			
-.004	-.005	-.006	3.59 / .84	1.05 / .16	.23 / .05	.17 / .04	.17 / .04	.18 / .04	.29 / .06	1.07 / .16	3.75 / .68	-.007	-.005	-.004
			3.66 / 1.11	.85 / 1.36	.23 / .06	.19 / .05	.18 / .04	.19 / .04	.30 / .06	1.02 / .16	3.74 / .71			
			3.75 / 1.18	.93 / .20	.33 / .06	.29 / .04	.27 / .04	.29 / .06	.48 / .10	1.40 / .25	3.82 / .76			
			3.86 / .95	2.65 / .55	1.42 / .22	1.30 / .20	1.09 / .15	1.25 / .22	1.43 / .36	2.96 / .93	3.87 / .85			
			3.96 / .85	3.82 / .98	3.71 / 1.17	3.66 / 1.52	3.78 / 1.72	3.61 / 1.40	3.63 / 1.20	3.82 / 1.11	3.95 / .84			
							-.0002							
							-.001							
							-.001							

TABLE I.I

GRAVSAT - GEOPAUSE (COPLANAR)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL] = (.001M,.001M,.001M)
HIGH SATELLITE STATE, σ [HCL] = (.001M,.001M,.001M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							-.104							
							-.159							
							-.277							
							-.638							
							-2.49							
					27.42/27.42	19.66/19.66	19.88/19.88	19.33/19.33	41.07/41.07					
					13.18/13.18	7.65/7.65	7.09/7.09	9.32/9.32	20.15/20.15					
-.059	-.069	-.089	-.167	-1.02	12.58/12.58	4.58/4.58	4.55/4.54	5.43/5.43	16.13/16.13	-1.07	-.236	-.115	-.080	-.064
					20.26/20.26	7.58/7.58	7.46/7.46	7.80/7.80	23.64/23.64					
					25.92/25.92	21.53/21.53	19.79/19.79	17.00/17.00	43.18/43.18					
							-2.38							
							-.517							
							-.220							
							-.128							
							-.084							

TABLE 1.11 GRAVSAT - GEOPAUSE (COPLANAR)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
25 DENSITY BLOCKS, σ = 50 MGAL
LOW SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)
HIGH SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
200 DENSITY BLOCKS, σ = 50 MGAL

							.008							
							.025							
							.063							
							.167							
				197.23/197.15	47.94/47.89	22.61/22.59	16.19/16.16	20.02/20.00	13.84/13.74	69.97/69.88				
				61.19/61.13	3.93/3.91	2.57/2.56	2.23/2.22	2.23/2.22	1.63/1.56	15.00/14.87				
				18.94/18.88	1.61/1.59	.90/.88	.84/.83	.87/.85	1.46/1.43	11.81/11.75				
-.061	-.078	-.103	-.119	12.47/12.44	1.90/1.88	1.04/1.03	.76/.74	.92/.90	1.32/1.29	9.95/9.90	-.145	-.112	-.082	-.062
				8.92/8.88	1.53/1.51	1.43/1.42	.89/.87	1.12/1.11	1.69/1.67	10.95/10.91				
				10.30/10.26	1.80/1.77	2.77/2.75	2.27/2.26	2.66/2.65	4.70/4.67	13.60/13.55				
				67.95/67.91	11.47/11.36	26.16/26.14	12.46/12.42	23.11/23.08	19.35/19.31	84.41/84.38				
							.179							
							.035							
							.017							
							-.0002							

TABLE I.III

GRAVSAT - GEOPAUSE (COPLANAR)

A POSTERIORI DENSITY SIGMAS

21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:

49 DENSITY BLOCKS, σ = 50 MGAL

LOW SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)

HIGH SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:

176 DENSITY BLOCKS, σ = 50 MGAL

							.005							
							-.006							
							-.016							
			75.78/62.07	52.50/34.47	28.70/12.61	48.99/43.20	26.54/17.04	59.22/55.00	32.86/21.78	43.44/24.61	86.42/74.82			
			47.95/24.74	12.71/1.86	6.09/4.89	3.68/2.96	2.41/1.57	4.79/4.26	3.31/1.36	14.09/8.89	48.01/22.45			
			46.37/29.14	6.27/4.02	.78/.46	.43/.24	.43/.25	.48/.31	.77/.42	5.44/2.75	44.98/28.49			
			46.69/38.56	4.72/3.47	.68/.49	.36/.22	.30/.17	.37/.22	.62/.37	4.73/3.14	41.39/28.04			
.007	.002	-.005	34.62/24.42	4.06/3.17	.58/.38	.34/.21	.33/.22	.37/.24	.61/.36	4.21/2.96	37.15/23.43	-.008	-.0001	.006
			36.35/27.15	3.51/2.64	.58/.38	.34/.18	.31/.18	.36/.20	.63/.33	4.18/3.02	39.89/25.61			
			45.92/35.60	4.45/3.46	.65/.39	.48/.30	.49/.36	.52/.32	.86/.35	5.30/3.23	43.55/25.48			
			48.62/28.50	14.72/11.59	3.18/1.68	2.72/1.78	4.79/4.35	2.51/1.59	4.26/1.51	13.94/8.53	46.61/20.38			
			87.68/76.43	38.65/18.47	31.90/15.28	24.36/14.89	50.77/46.60	22.40/12.55	26.02/13.47	37.24/17.03	92.44/82.12			
							-.058							
							-.047							
							-.029							

TABLE I.IV

GRAVSAT - GEOPAUSE (COPLANAR)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 50 MGAL
LOW SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)
HIGH SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 50 MGAL

							-.002							
							-.002							
							-.0006							
			4.00/1.50	3.93/1.16	3.65/1.29	3.61/1.55	3.17/1.28	3.58/1.39	3.37/1.30	3.71/1.13	3.96/.86			
			3.96/1.15	3.05/.67	1.38/.53	.77/.08	.92/.19	.95/.07	1.23/.30	2.96/.74	3.92/.89			
			3.92/1.18	1.77/.29	.41/.05	.25/.05	.26/.04	.29/.04	.50/.10	1.70/.32	3.85/.88			
			3.82/1.14	1.39/.14	.31/.06	.20/.05	.20/.04	.23/.04	.41/.09	1.44/.22	3.81/.79			
-.004	-.005	-.005	3.59/.97	1.07/.18	.26/.06	.19/.05	.20/.04	.23/.04	.40/.08	1.32/.23	3.75/.69	-.005	-.004	-.004
			3.72/1.18	.86/.18	.25/.06	.20/.05	.21/.04	.24/.05	.41/.09	1.22/.18	3.75/.70			
			3.76/1.32	.93/.23	.31/.06	.27/.06	.27/.05	.32/.06	.54/.12	1.37/.28	3.86/.79			
			3.88/1.06	2.54/.62	.79/.19	1.07/.14	.88/.10	1.12/.15	1.37/.40	2.89/.79	3.92/.98			
			3.96/.90	3.51/.92	3.25/1.00	3.16/1.42	3.59/1.64	2.87/1.13	3.34/1.20	3.80/1.36	3.97/.98			
							-.0005							
							-.002							
							-.002							

TABLE I.V

GRAVSAT - GEOPAUSE (COPLANAR)

A POSTERIORI DENSITY SIGMAS

21 SHORT DATA ARCS (DATA NOISE = .05CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (.02M,.14M,4.78M)
HIGH SATELLITE STATE, σ [HCL]= (.03M,.62M,.65M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							-.0006							
							.0003							
							.003							
			3.97 / .34	3.92 / .49	3.82 / .68	3.80 / .80	3.77 / .83	3.79 / .76	3.83 / .62	3.90 / .51	3.96 / .38			
			3.95 / .35	3.58 / .35	1.89 / .23	1.59 / .18	1.62 / .18	1.58 / .24	1.88 / .22	3.48 / .59	3.93 / .45			
			3.92 / .31	2.43 / .32	.67 / .08	.41 / .05	.41 / .05	.45 / .06	.77 / .13	2.45 / .40	3.88 / .45			
			3.87 / .33	1.71 / .20	.49 / .07	.32 / .05	.33 / .04	.36 / .05	.60 / .09	1.65 / .31	3.85 / .36			
-.004	-.005	-.006	3.81 / .45	1.37 / .21	.42 / .07	.30 / .05	.30 / .04	.34 / .05	.61 / .08	1.52 / .30	3.83 / .37	-.005	-.005	-.004
			3.85 / .38	1.16 / .18	.42 / .07	.34 / .05	.33 / .05	.37 / .05	.65 / .08	1.40 / .29	3.83 / .40			
			3.87 / .37	1.25 / .15	.53 / .07	.46 / .05	.43 / .05	.47 / .07	.87 / .08	1.76 / .26	3.90 / .41			
			3.93 / .36	3.08 / .40	2.07 / .21	1.61 / .20	1.51 / .16	1.67 / .22	1.79 / .25	3.61 / .56	3.94 / .42			
			3.96 / .33	3.88 / .49	3.80 / .65	3.76 / .78	3.79 / .80	3.76 / .76	3.81 / .63	3.91 / .50	3.97 / .36			
							.003							
							.00007							
							-.0010							

TABLE I.VI

GRAVSAT - GEOPAUSE (COPLANAR)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (2M,5M,10M)
HIGH SATELLITE STATE, σ [HCL]= (.4M,1M,2M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							.004							
							.008							
							.018							
			3.99 / .07	3.96 / .13	3.90 / .25	3.87 / .32	3.86 / .34	3.87 / .31	3.90 / .22	3.96 / .14	3.99 / .07			
			3.99 / .05	3.67 / .08	2.22 / .20	1.94 / .18	2.00 / .18	2.08 / .19	2.34 / .20	3.75 / .14	3.98 / .06			
			3.97 / .06	2.81 / .12	1.13 / .11	1.05 / .04	.99 / .07	1.05 / .07	1.42 / .16	2.73 / .15	3.95 / .07			
			3.93 / .12	2.04 / .17	1.00 / .08	.91 / .05	.89 / .05	.93 / .05	1.22 / .13	1.91 / .25	3.90 / .15			
-.006	-.008	-.011	3.93 / .08	1.90 / .13	.94 / .08	.88 / .05	.86 / .06	.88 / .05	1.20 / .13	1.82 / .25	3.88 / .16	-.009	-.007	-.005
			3.90 / .15	1.68 / .19	.93 / .07	.89 / .06	.88 / .05	.91 / .06	1.30 / .11	1.78 / .26	3.90 / .14			
			3.93 / .11	1.85 / .18	1.04 / .08	.99 / .07	.98 / .06	1.04 / .08	1.54 / .10	1.99 / .24	3.95 / .08			
			3.98 / .05	3.35 / .22	2.34 / .15	1.99 / .19	1.89 / .17	2.03 / .15	2.48 / .20	3.81 / .12	3.98 / .06			
			3.99 / .08	3.94 / .16	3.89 / .26	3.85 / .35	3.86 / .35	3.87 / .31	3.90 / .23	3.96 / .14	3.99 / .07			
							.010							
							.004							
							.002							

TABLE I.VII

GRAVSAT - GEOPAUSE (COPLANAR)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (10M,25M,50M)
HIGH SATELLITE STATE, σ [HCL]= (2M,5M,10M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							.006							
							.012							
							.026							
			4.00 .03	3.98 .07	3.93 .14	3.91 .18	3.90 .20	3.91 .18	3.93 .13	3.97 .07	3.99 .03			
			3.99 .02	3.70 .07	2.42 .16	2.22 .13	2.25 .14	2.33 .13	2.66 .14	3.82 .08	3.99 .02			
			3.97 .05	2.96 .11	1.39 .13	1.39 .05	1.29 .09	1.34 .09	1.73 .14	2.79 .14	3.96 .06			
			3.95 .08	2.14 .14	1.26 .09	1.22 .05	1.17 .07	1.19 .06	1.43 .13	2.05 .20	3.92 .10			
-.006	-.008	-.013	3.95 .05	2.00 .11	1.21 .09	1.17 .06	1.14 .07	1.16 .07	1.34 .14	2.00 .17	3.91 .10	-.011	-.008	-.005
			3.92 .10	1.81 .16	1.22 .07	1.16 .07	1.15 .07	1.19 .07	1.43 .13	2.03 .18	3.92 .10			
			3.95 .08	2.00 .18	1.33 .10	1.25 .10	1.27 .08	1.33 .10	1.72 .13	2.14 .20	3.96 .06			
			3.99 .02	3.49 .14	2.57 .12	2.21 .13	2.16 .12	2.25 .10	2.78 .12	3.85 .07	3.99 .03			
			3.99 .04	3.96 .09	3.92 .15	3.90 .20	3.90 .20	3.91 .18	3.93 .13	3.97 .07	3.99 .03			
							-017							
							.008							
							.004							

TABLE I.VIII GRAVSAT - GEOPAUSE (COPLANAR)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (20M,50M,100M)
HIGH SATELLITE STATE, σ [HCL]= (4M,10M,20M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							-.0009							
							.00008							
							.0003							
			3.97 / .26	3.93 / .40	3.83 / .59	3.80 / .70	3.78 / .74	3.80 / .67	3.85 / .50	3.92 / .37	3.97 / .25			
			3.96 / .26	3.60 / .27	1.94 / .24	1.61 / .19	1.66 / .19	1.62 / .28	1.96 / .25	3.56 / .37	3.95 / .28			
			3.93 / .21	2.53 / .29	.71 / .07	.44 / .05	.45 / .05	.48 / .06	.90 / .12	2.57 / .28	3.91 / .29			
			3.88 / .28	1.76 / .21	.53 / .07	.34 / .05	.37 / .04	.39 / .06	.75 / .08	1.71 / .30	3.86 / .29			
-.004	-.005	-.006	3.83 / .41	1.43 / .21	.46 / .067	.33 / .048	.33 / .04	.38 / .05	.76 / .07	1.59 / .29	3.83 / .33	-.006	-.005	-.004
			3.86 / .35	1.21 / .19	.45 / .07	.38 / .05	.35 / .05	.43 / .06	.82 / .06	1.46 / .30	3.84 / .32			
			3.89 / .25	1.31 / .16	5.67 / .08	.50 / .06	.45 / .05	.55 / .08	1.03 / .07	1.80 / .27	3.92 / .25			
			3.94 / .26	3.11 / .35	2.10 / .22	1.65 / .20	1.53 / .18	1.72 / .23	1.90 / .24	3.69 / .35	3.95 / .26			
			3.96 / .26	3.89 / .42	3.82 / .59	3.78 / .72	3.79 / .72	3.79 / .66	3.84 / .52	3.93 / .36	3.97 / .24			
							-.006							
							.002							
							.0003							

TABLE I.IX GRAVSAT - GEOPAUSE (COPLANAR)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (2M,5M,10M)
HIGH SATELLITE STATE, σ [HCL]= (20M,50M,100M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							.006							
							.011							
							.026							
			4.00 .02	3.98 .05	3.94 .11	3.92 .14	3.92 .15	3.92 .15	3.94 .11	3.98 .06	4.00 .03			
			3.99 .02	3.74 .07	2.58 .14	2.33 .12	2.41 .12	2.45 .12	2.83 .13	3.84 .08	3.99 .02			
			3.97 .05	3.07 .11	1.56 .13	1.45 .05	1.41 .10	1.41 .09	1.82 .13	2.88 .14	3.96 .05			
			3.95 .07	2.18 .15	1.36 .10	1.27 .05	1.25 .07	1.25 .07	1.48 .12	2.12 .19	3.93 .09			
-.005	-.007	-.012	3.95 .04	2.03 .11	1.27 .09	1.21 .06	1.19 .07	1.21 .07	1.38 .13	2.03 .16	3.93 .08	-.010	-.007	-.005
			3.92 .09	1.86 .16	1.29 .08	1.22 .07	1.20 .07	1.26 .08	1.46 .13	2.13 .16	3.93 .09			
			3.95 .08	2.10 .19	1.44 .10	1.36 .10	1.34 .08	1.44 .11	1.78 .12	2.28 .20	3.96 .06			
			3.99 .02	3.54 .13	2.65 .11	2.35 .11	2.27 .10	2.43 .09	2.92 .11	3.88 .06	3.99 .02			
			3.99 .03	3.97 .07	3.93 .12	3.91 .16	3.91 .16	3.92 .14	3.94 .11	3.98 .06	4.00 .03			
							.019							
							.009							
							.005							

TABLE I.X

GRAVSAT - GEOPAUSE (COPLANAR)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (20M,50M,100M)
HIGH SATELLITE STATE, σ [HCL]= (200M,500M, 1000M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							.006							
							.012							
							.028							
			4.00 / .009	3.99 / .029	3.96 / .070	3.94 / .090	3.94 / .092	3.94 / .088	3.95 / .070	3.98 / .039	4.00 / .013			
			4.00 / .014	3.79 / .053	2.81 / .124	2.67 / .093	2.71 / .086	2.75 / .091	3.10 / .115	3.88 / .053	4.00 / .009			
			3.98 / .041	3.25 / .087	1.80 / .123	1.72 / .068	1.66 / .101	1.67 / .098	1.99 / .102	2.94 / .128	3.97 / .046			
			3.96 / .048	2.36 / .117	1.53 / .090	1.46 / .064	1.44 / .078	1.45 / .075	1.60 / .101	2.31 / .156	3.94 / .062			
-.004	-.005	-.008	3.96 / .030	2.12 / .085	1.44 / .088	1.39 / .072	1.38 / .077	1.40 / .075	1.46 / .118	2.15 / .109	3.95 / .037	-.008	-.005	-.004
			3.94 / .060	2.02 / .119	1.47 / .083	1.41 / .081	1.41 / .077	1.46 / .079	1.54 / .121	2.30 / .117	3.94 / .061			
			3.96 / .056	2.34 / .164	1.73 / .108	1.62 / .104	1.62 / 091	1.68 / .111	1.89 / .113	2.47 / .181	3.96 / .048			
			3.99 / .017	3.65 / .075	2.89 / .102	2.67 / .083	2.61 / .077	2.72 / .085	3.15 / .096	3.91 / .040	4.00 / .011			
			4.00 / .013	3.98 / .040	3.96 / .072	3.94 / .095	3.94 / .094	3.95 / .086	3.96 / .068	8.99 / .035	4.00 / .013			
							.025							
							.011							
							.006							

TABLE I.XI GRAVSAT - GEOPAUSE (COPLANAR)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (40M,100M,200M)
HIGH SATELLITE STATE, σ [HCL]= (400M,1000M,2000M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							-.0006							
							.0003							
							.003							
			4.00/.59	3.96/.77	3.88/.96	3.87/1.08	3.86/1.16	3.85/1.02	3.93/1.10	4.00/1.01	4.01/.73			
			4.00/.71	3.65/.80	2.02/.74	1.68/.56	1.75/.70	1.71/.68	2.04/.82	3.78/1.59	4.02/.99			
			3.98/.75	2.68/1.16	.77/.40	.49/.26	.54/.35	.53/.28	1.03/.70	2.83/1.47	3.99/1.01			
			3.93/.78	1.83/.68	.58/.33	.40/.25	.44/.30	.44/.25	.86/.62	1.83/.87	3.89/.69			
-.004	-.005	-.006	3.92/1.04	1.48/.61	.51/.30	.41/.29	.39/.26	.45/.30	.88/.65	1.69/.79	3.86/.61	-.005	-.005	-.004
			3.89/.71	1.29/.58	.50/.29	.49/.36	.41/.25	.53/.39	.94/.69	1.56/.75	3.88/.74			
			4.09/1.39	1.38/.59	.63/.35	.63/.44	.52/.30	.68/.49	1.17/.79	1.87/.70	3.99/.95			
			4.09/1.19	3.26/1.13	2.22/.83	1.78/.78	1.62/.61	1.91/.96	2.02/.96	3.89/1.55	4.03/.96			
			3.99/.61	4.06/1.29	4.15/1.79	4.16/1.94	3.99/1.23	4.04/1.66	4.11/1.66	3.99/.94	4.01/.70			
							.003							
							.00007							
							.0010							

TABLE I.XII

GRAVSAT - GEOPAUSE (COPLANAR)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (2M,5M,10M)
HIGH SATELLITE STATE, σ [HCL]= (20M,50M,100M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

			-.02	-.02	-.01	.02	.04	.01	-.01	-.02	-.02			
			-.03	-.01	.02	.06	-.15	.07	.007	-.002	-.03			
			-.05	.05	.05	-.02	.39	-.04	.07	.01	-.04			
			-.05	.04	.01	.30	-.57	.31	-.002	.08	-.05			
			-.03	.004	.13	-.27	1	-.38	.21	-.02	-.05			
			-.04	.09	.01	.31	-.57	.30	-.01	.10	-.05			
			-.05	.04	.08	-.04	.38	-.02	.07	.01	-.04			
			-.03	.003	.02	.05	-.14	.02	.02	-.009	-.03			
			-.02	-.01	-.007	.01	.05	.03	0	-.01	-.01			

TABLE I.XIII GRAVSAT - GEOPAUSE (COPLANAR)

CORRELATIONS OF ADJUSTED BLOCKS WITH CENTER BLOCK
21 SHORT DATA ARCS (DATA NOISE = .05 CM.SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)
HIGH SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

			-.22	-.12	.17	-.14	.41	-.22	.26	-.14	-.25			
			-.11	.12	.13	.25	-.19	.33	.13	.16	-.08			
			-.02	.17	.33	.47	.72	.35	.39	.20	-.09			
			-.29	.35	.45	.69	.47	.68	.48	.35	-.25			
			.10	.30	.48	.51	1	.53	.50	.34	-.13			
			.08	.13	.46	.62	.50	.62	.43	.35	-.17			
			-.37	.44	.39	.54	.77	.58	-.05	.35	-.30			
			.09	-.37	.30	-.27	-.50	-.28	.27	-.27	-.01			
			.43	.20	-.10	.25	.53	.04	.21	-.22	.40			

TABLE I.XIV GRAVSAT - GEOPAUSE (COPLANAR)

CORRELATIONS OF ADJUSTED BLOCKS WITH CENTER BLOCK
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 50 MGAL
LOW SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)
HIGH SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 50 MGAL

			-.02	-.02	-.01	.02	.05	.01	-.01	-.01	-.01			
			-.03	-.03	.02	-.05	.12	-.08	.02	-.02	-.02			
			-.03	-.04	-.13	-.27	.65	-.28	.13	-.06	-.02			
			-.03	-.04	.12	-.24	.48	-.22	.11	-.04	-.01			
			-.02	-.07	.21	-.47	1	-.40	.17	-.09	-.01			
			-.02	-.03	.15	-.28	.45	-.19	.11	-.03	-.03			
			-.03	-.05	.16	-.33	.62	-.24	.13	-.06	-.04			
			-.02	-.02	.03	-.09	.09	-.06	.05	-.05	-.04			
			-.01	-.009	0	.02	.05	.03	-.01	-.03	-.03			

TABLE I.XV

GRAVSAT - GEOPAUSE (COPLANAR)

CORRELATIONS OF ADJUSTED BLOCKS WITH CENTER BLOCK
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (2M,5M,10M)
HIGH SATELLITE STATE, σ [HCL]= (.4M,1M,2M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							-.008							
							-.005							
							-.0004							
			71.80/57.31	50.76/31.23	32.65/20.76	49.81/43.12	25.91/14.46	51.44/46.36	33.19/21.44	44.44/27.43	92.81/82.88			
			51.01/29.69	14.23/9.82	4.64/2.50	3.77/2.97	2.35/1.23	4.38/3.74	3.86/2.10	21.92/17.81	52.95/34.19			
			45.62/27.84	5.99/4.27	.65/.35	.41/.24	.39/.22	.47/.33	.62/.28	5.53/1.73	39.84/21.46			
			44.77/30.76	5.39/4.54	.45/.28	.31/.22	.25/.15	.29/.20	.37/.13	4.08/1.82	36.17/20.42			
-.027	-.025	-.017	46.27/37.50	5.74/5.18	.46/.33	.29/.21	.28/.20	.28/.19	.38/.16	3.89/2.27	34.51/20.05	-.010	-.015	-.017
			36.95/28.20	4.49/3.96	.45/.31	.28/.19	.28/.19	.26/.15	.41/.19	3.51/2.01	37.05/23.75			
			40.11/29.43	3.70/2.89	.55/.35	.41/.22	.48/.35	.39/.15	.79/.40	5.14/1.60	39.56/21.55			
			50.74/34.68	14.71/12.59	3.04/1.59	2.56/1.36	4.89/4.41	2.66/1.69	4.87/3.32	21.75/17.64	53.03/33.13			
			89.59/79.46	36.99/18.33	31.42/17.02	26.75/18.88	43.07/38.23	25.39/16.35	24.80/10.04	34.60/16.02	89.56/79.41			
							-.018							
							-.023							
							-.023							

TABLE I.XVI

GRAVSAT - GEOPAUSE (30° Off-Plane)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 50 MGAL
LOW SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)
HIGH SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 50 MGAL

							-.013							
							-.011							
							-.003							
			51.00/21.04	48.35/21.48	38.87/18.00	38.93/19.04	36.89/20.23	40.49/23.96	37.36/16.42	44.61/16.53	50.55/17.99			
			49.31/21.00	13.40/3.95	6.39/2.60	4.07/1.71	4.23/1.98	4.56/2.31	5.06/1.39	20.25/6.77	47.26/15.53			
			46.08/22.62	7.19/3.53	1.33/.63	.70/.36	.65/.35	.61/.22	1.23/.61	9.94/2.51	41.72/17.40			
			43.24/21.51	5.93/2.44	1.18/.59	.58/.34	.49/.25	.48/.20	1.00/.46	8.63/2.79	38.14/18.70			
-.046	-.052	-.057	39.65/19.54	5.35/2.19	1.16/.60	.56/.34	.49/.26	.48/.23	.99/.43	7.80/3.66	35.81/19.49	.003	-.004	-.007
			33.95/21.88	4.49/2.56	1.07/.59	.54/.32	.50/.25	.55/.29	1.06/.46	7.65/4.40	37.84/22.43			
			40.83/25.68	5.32/3.46	1.06/.57	.67/.36	.65/.33	.70/.36	1.34/.45	8.86/4.48	41.00/19.63			
			47.28/26.40	10.45/5.71	4.23/1.11	3.69/1.45	4.46/2.71	3.66/1.02	5.00/1.45	22.43/8.67	46.51/15.14			
			51.19/24.20	42.42/18.18	36.54/15.25	33.76/18.01	41.51/27.44	34.08/16.03	37.04/15.60	44.41/20.64	49.32/16.05			
							-.0004							
							-.008							
							-.010							

TABLE I.XVII

GRAVSAT - GEOPAUSE (30° Off-Plane)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 50 MGAL
LOW SATELLITE STATE, σ [HCL]= (2M,5M,10M)
HIGH SATELLITE STATE, σ [HCL]= (.4M,1M,2M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 50 MGAL

							-.001							
							-.0009							
							-.0003							
			46.49/1.68	43.35/1.72	34.48/1.44	33.99/1.52	30.89/1.62	32.70/1.92	33.58/1.31	41.46/1.32	47.26/1.44			
			44.65/1.68	12.81/.32	5.84/.21	3.70/.14	3.75/.16	3.94/.19	4.87/.11	19.10/.54	44.65/1.24			
			40.18/1.81	6.27/.28	1.17/.05	.60/.03	.55/.03	.57/.02	1.06/.05	9.62/.20	37.95/1.39			
			37.55/1.72	5.40/.20	1.03/.05	.47/.03	.42/.02	.44/.02	.89/.04	8.17/.22	33.28/1.50			
-.004	-.004	-.005	34.53/1.56	4.89/.17	.99/.05	.44/.03	.41/.02	.43/.02	.90/.03	6.90/.29	30.08/1.56	.0002	-.0003	-.0006
			26.02/1.75	3.70/.20	.90/.05	.44/.03	.44/.02	.47/.02	.96/.04	6.27/.35	30.53/1.79			
			31.80/2.05	4.05/.28	.90/.05	.56/.03	.56/.03	.60/.03	1.27/.04	7.65/.36	36.03/1.57			
			39.28/2.11	8.77/.46	4.09/.09	3.40/.12	3.55/.22	3.52/.08	4.79/.12	20.70/.69	43.99/1.21			
			45.15/1.94	38.35/1.45	33.23/1.22	28.58/1.44	31.23/2.20	30.10/1.28	33.62/1.25	39.35/1.65	46.65/1.28			
							-.00003							
							-.0006							
							-.0008							

TABLE I.XVIII

GRAVSAT - GEOPAUSE (30° Off-Plane)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS σ = 50 MGAL
LOW SATELLITE STATE, σ [HCL]= (2M,5M,10M)
HIGH SATELLITE STATE, σ [HCL]= (.4M,1M,2M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							.0008							
							.0002							
							.0005							
			3.97 .24	3.93 .40	3.83 .56	3.81 .65	3.80 .69	3.82 .62	3.85 .52	3.90 .44	3.96 .30			
			3.95 .22	3.34 .25	2.13 .41	1.63 .22	1.78 .29	1.67 .27	2.10 .39	3.35 .73	3.91 .39			
			3.93 .18	1.77 .30	.73 .13	.43 .06	.44 .05	.46 .05	.70 .06	2.38 .62	3.87 .40			
			3.92 .13	1.31 .29	.59 .13	.34 .06	.35 .05	.37 .04	.60 .04	1.72 .43	3.85 .38			
-.0007	-.0009	-.001	3.92 .13	1.18 .27	.530 .13	.312 .05	.34 .05	.35 .04	.60 .04	1.68 .42	3.82 .43	.0005	.0003	.0002
			3.87 .23	1.13 .27	.52 .12	.34 .06	.38 .05	.38 .04	.62 .04	1.63 .396	3.81 .46			
			3.87 .36	1.37 .27	.59 .12	.46 .06	.48 .05	.49 .06	.75 .08	2.07 .47	3.83 .49			
			3.90 .38	2.62 .45	2.02 .28	1.69 .27	1.54 .20	1.81 .31	1.90 .28	3.34 .81	3.88 .46			
			3.95 .35	3.87 .495	3.81 .62	3.78 .70	3.81 .67	3.79 .60	3.84 .49	3.91 .42	3.95 .31			
							.0009							
							0004							
							.0002							

TABLE I.XIX

GRAVSAT - GEOPAUSE (30° Off-Plane)

A POSTERIORI DENSITY SIGMAS

21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (2M,5M,10M)
HIGH SATELLITE STATE, σ [HCL]= (.4M,1M,2M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							-.033							
							-.063							
							-.139							
							-.405							
							-1.90							
					42.22 42.22	26.14 26.14	26.79 26.79	26.79 26.79	55.04 55.04					
					23.82 23.82	10.43 10.43	7.98 7.98	11.14 11.14	19.05 19.05					
-.212	-.277	-.390	-.667	-1.92	27.40 27.40	9.04 9.04	7.58 7.58	8.39 8.39	12.42 12.42	1.25	1.13	.692	.451	.312
					22.25 22.25	8.35 8.35	10.02 10.01	7.53 7.53	20.79 20.79					
					49.26 49.26	26.34 26.34	29.51 29.51	21.10 21.10	51.26 51.26					
							-3.53							
							-1.06							
							-.469							
							-.259							
							-.159							

TABLE I.XX

GRAVSAT - GEOPAUSE (45° Off-Plane)

A POSTERIORI DENSITY SIGMAS

21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:

25 DENSITY BLOCKS, σ = 50 MGAL

LOW SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)

HIGH SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:

200 DENSITY BLOCKS, σ = 50 MGAL

							-.014							
							-.007							
							.017							
							.104							
				54.81/54.65	17.39/17.27	17.03/16.99	19.62/19.59	18.70/18.67	11.51/11.36	53.52/53.41				
				14.21/14.03	2.41/2.37	2.20/2.18	1.54/1.50	1.79/1.76	1.80/1.73	19.61/19.52				
				14.32/14.30	1.87/1.85	1.30/1.29	.79/.76	.89/.87	1.13/1.09	20.11/20.08				
-.068	-.081	-.097	-.099	10.78/10.76	2.40/2.39	.78/.75	.68/.66	.65/.62	1.14/1.11	19.96/19.94	.072	.031	.016	.007
				13.35/13.33	1.41/1.39	1.20/1.18	1.10/1.08	1.10/1.08	1.30/1.27	19.78/19.76				
				10.30/10.26	2.36/2.33	1.89/1.86	2.45/2.43	2.11/2.09	2.12/2.07	23.67/23.64				
				54.11/54.06	12.17/12.06	24.66/24.63	13.82/13.78	26.74/26.71	10.93/10.84	55.91/55.86				
							.247							
							.111							
							.052							
							.025							

TABLE I.XXI GRAVSAT - GEOPAUSE (45° Off-Plane)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
49 DENSITY BLOCKS, σ = 50 MGAL
LOW SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)
HIGH SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
176 DENSITY BLOCKS, σ = 50 MGAL

							-.013							
							-.011							
							-.005							
			67.15 / 50.92	49.84 / 29.25	29.79 / 15.43	43.32 / 35.14	24.97 / 12.31	43.62 / 37.13	30.00 / 16.20	43.57 / 26.00	87.04 / 76.04			
			52.69 / 32.27	15.34 / 11.48	4.56 / 2.00	3.55 / 2.47	2.41 / 1.00	3.95 / 3.01	3.54 / 1.81	23.07 / 19.32	54.30 / 36.20			
			48.05 / 30.93	7.55 / 5.85	.69 / .38	.43 / .24	.45 / .26	.50 / .34	.67 / .34	7.11 / 3.90	39.02 / 15.95			
			48.31 / 34.86	7.63 / 6.74	.48 / .34	.32 / .23	.27 / .16	.31 / .21	.41 / .20	5.55 / 3.47	35.78 / 16.32			
-.026	-.023	-.015	52.52 / 43.31	7.92 / 7.23	.48 / .36	.30 / .22	.28 / .19	.30 / .21	.41 / .24	4.99 / 3.32	35.19 / 18.42	-.005	-.011	-.014
			34.68 / 22.70	4.74 / 3.95	.44 / .32	.29 / .20	.30 / .20	.29 / .18	.45 / .27	4.08 / 2.40	36.41 / 21.55			
			38.22 / 25.68	3.53 / 2.39	.60 / .40	.45 / .26	.52 / .37	.43 / .19	.85 / .57	5.36 / 1.86	38.97 / 20.50			
			51.25 / 34.49	15.97 / 13.38	3.23 / 1.66	2.45 / .99	4.61 / 3.99	2.47 / 1.03	4.02 / 2.65	20.00 / 16.19	54.28 / 36.12			
			85.34 / 74.37	37.00 / 18.90	31.50 / 17.32	27.09 / 19.30	39.62 / 34.12	25.89 / 16.65	25.33 / 10.37	33.51 / 14.26	76.18 / 63.51			
							-.012							
							-.017							
							-.019							

TABLE I.XXII GRAVSAT - GEOPAUSE (45° Off-Plane)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 50 MGAL
LOW SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)
HIGH SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 50 MGAL

SET II:

Solution Set II investigates the "high-low" GRAVSAT/GEOSTATIONARY configuration as described in Section 6.1. This study is designed to compare the efficiency of the geostationary high satellite concept for gravity find structure recovery with the "low-low" configuration of report [A] (Set VI) and the high GEOPAUSE satellite of Set I.

Table II.I displays the most favorable case for the high GEOSTATIONARY with two "buffer layers", a realistic 4 mgal surface density uncertainty and nearly perfect knowledge of the epoch states, while Table II.II shows the same situation with 50 mgal uncertainty for the density blocks. Comparison with solution Set I shows that the GEOPAUSE and GEOSTATIONARY spacecraft have very similar characteristics with regard to recovery of gravity fine structure. The geostationary configuration has a slight advantage when the density block a priori uncertainties are small (4 mgal) and the GEOPAUSE configuration is more efficient when the a priori uncertainties are very large (50 mgal).

The case displayed in Table II.I, assuming error free satellite epoch state errors can be compared with results obtained by Hajela [8] for a "high-low" configuration with the low satellite at 250 kn. altitude. His solution set 250-5-2 for the estimation of 24 5° x 5° equal-area gravity anomaly blocks, with data over the interior 12 blocks, yielded a R.M.S. standard deviation of 3.4 mgal for the interior anomaly blocks. Hajela assumed a data noise of .08 cm/sec, a data interval of 10 seconds, and two passes per block. His result can be compared to the result of the center density block of Table II.I by the adjustment

$$\left(\frac{1}{2\pi}\right)\cdot\left(3.4 \text{ mgal}\right)\cdot\left(\frac{.05 \text{ cm/sec}}{.08 \text{ cm/sec}}\right)\cdot\left(\frac{1}{\sqrt{2}}\right)\cdot\left(\frac{1}{\sqrt{2}}\right) = .17 \text{ mgal}$$

This compares very closely with the .15 mgal found in Table II.I.

As in Set I, to determine the character of the density block recovery as a function of epoch state a priori errors, Tables II.III-II.VII utilize multiples of (2m, 5m, 10m) and (20m, 50m, 100m) in HCL coordinates for the GRAVSAT and GEOSTATIONARY position uncertainties, respectively. These uncertainties, with a 4 mgal uncertainty for the density blocks results in an error of .27 mgal in the recovery of the center block, while increasing the state a priori by a factor of 20 results in an error of 1.36 mgal.

Tables II.VIII and II.IX, together with II.IV demonstrate the effect of changing the magnitude of the a priori uncertainty for the unadjusted blocks from 50 mgal to 4 mgal while keeping the large a priori value of 50 mgal for the adjusted blocks. This is to be compared with Tables I.XIII-I.XV.

The effect of high satellite unadjusted state errors contaminating the recovery is observed in Table II.X. Here geostationary state errors of (20m, 50m, 100m) and 4 mgal uncertainty in these density blocks result in an error in the center adjusted block of .29 mgal, with an aliasing component of .14 mgal. This is a slightly degraded solution relative to that of Table II.IV. The recovery of gravitational fine structure from SST range-rate is seen to be insensitive to the errors of the high satellite when these errors are reasonably small (order of tens of meters).

The effect of reduced data noise is displayed in Table II.XI where the noise sigma is set to .01 cm/sec, a factor of fine improvement, resulting in a total recovery error of .22 mgal. This is to be compared with the solution of Table II.IV with a noise sigma of .05 cm/sec, where we found a recovery error of .27 mgal. The aliasing component of error is the same in both instances, showing the dominating error source to be the a priori uncertainties in the adjusted parameters.

The GRAVSAT satellite has been presented in these studies as a drag free satellite with a surface force compensation system and thrusters to maintain an orbit due to only gravitational forces. However, following the error analysis case study of Table II.V, drag and drag-rate parameters were added to the set of unadjusted parameters for the low satellite for each of the twenty-one arcs. The error analysis displayed virtually no impact of the drag parameters (even with large a priori errors) on the recovery of the density blocks. This result is strongly dependent, of course, on the assumed use of short (approximately 10 minutes), independent data arcs with reasonably small a priori uncertainties of the low satellite epoch state vector.

The correlations of the adjusted blocks with the center block for the error analysis studies of Tables II.I and II.IV are presented in Tables II.XIII and II.XIV, respectively. The correlations display good resolvability in the recovery of the 5° x 5° blocks, and compare very closely with the correlations for the corresponding error analysis studies of the GRAVSAT/GEOPAUSE configuration in Set I.

							-.002							
							-.003							
							-.002							
			3.97/.41	3.93/.60	3.84/.88	3.81/1.04	3.80/1.04	3.82/1.05	3.86/.97	3.91/.76	3.96/.54			
			3.95/.38	3.35/.39	1.42/.22	1.19/.16	1.18/.18	1.19/.16	1.40/.29	3.05/.53	3.91/.60			
			3.95/.28	1.57/.25	.37/.07	.27/.06	.27/.05	.27/.06	.35/.07	1.36/.23	3.90/.55			
			3.95/.19	1.19/.23	.25/.07	.17/.05	.16/.04	.17/.05	.25/.06	1.04/.14	3.90/.47			
-.004	-.005	-.006	3.96/.11	.94/.22	.23/.07	.16/.05	.15/.04	.16/.04	.22/.06	.90/.15	3.9/.47	-.005	-.004	-.003
			3.96/.16	.76/.21	.22/.08	.17/.05	.17/.04	.17/.05	.22/.06	.82/.15	3.88/.56			
			3.95/.26	.97/.23	.32/.08	.28/.06	.27/.05	.28/.06	.32/.07	.92/.18	3.87/.65			
			3.95/.35	2.52/.26	1.43/.20	1.21/.16	1.20/.17	1.21/.17	1.36/.23	2.24/.30	3.90/.63			
			3.97/.39	3.91/.58	3.83/.89	3.81/1.04	3.81/1.04	3.82/1.04	3.86/.94	3.91/.72	3.96/.54			
							-.003							
							-.003							
							-.002							

TABLE II.I

GRAVSAT - GEOSTATIONARY

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)
HIGH SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, = 4 MGAL

							-.027							
							-.032							
							-.028							
			50.76 / 18.29	46.05 / 15.38	35.30 / 17.13	32.78 / 20.33	31.19 / 18.38	32.56 / 20.12	34.22 / 15.87	44.88 / 12.76	50.53 / 17.14			
			49.16 / 17.55	12.76 / 2.87	4.54 / 2.44	3.08 / 1.97	3.26 / 2.23	3.71 / 2.55	4.68 / 2.48	11.43 / 3.75	47.61 / 19.19			
			46.66 / 15.55	6.42 / 2.24	.88 / .59	.66 / .51	.62 / .50	.66 / .54	.81 / .60	5.01 / 2.14	44.74 / 19.37			
			45.33 / 15.27	5.48 / 1.40	.76 / .52	.53 / .42	.46 / .38	.48 / .40	.60 / .46	4.42 / 2.09	44.49 / 20.37			
-.040	-.044	-.047	43.24 / 12.94	5.12 / 1.28	.80 / .53	.54 / .43	.46 / .38	.44 / .37	.52 / .41	4.34 / 2.15	43.63 / 20.32	-.025	-.030	-.029
			44.60 / 13.17	5.28 / 1.43	.85 / .57	.54 / .41	.46 / .38	.48 / .40	.57 / .45	4.05 / 2.04	44.38 / 21.04			
			43.62 / 15.11	5.57 / 1.82	.94 / .63	.67 / .51	.63 / .51	.66 / .54	.79 / .61	4.19 / 1.78	43.17 / 18.39			
			46.99 / 18.72	9.33 / 3.67	3.75 / 1.71	3.02 / 1.87	3.22 / 2.10	3.63 / 2.34	4.33 / 2.27	9.39 / 4.19	46.04 / 19.05			
			50.50 / 18.07	42.91 / 14.91	34.55 / 14.69	32.89 / 20.45	31.24 / 17.84	32.65 / 19.41	34.83 / 14.37	43.39 / 14.02	50.50 / 17.43			
							-.026							
							-.032							
							-.028							

TABLE 11.11

GRAVSAT - GEOSTATIONARY

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 50 MGAL
LOW SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)
HIGH SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 50 MGAL

							-.002							
							-.003							
							-.003							
			3.99 .07	3.97 .10	3.95 .14	3.94 .18	3.93 .20	3.94 .21	3.94 .19	3.96 .15	3.99 .10			
			3.98 .08	3.50 .17	1.86 .23	1.67 .19	1.66 .18	1.66 .19	1.86 .22	3.40 .23	3.97 .12			
			3.96 .07	1.80 .25	.54 .10	.38 .07	.37 .06	.37 .07	.51 .08	1.68 .26	3.95 .12			
			3.96 .06	1.30 .22	.36 .08	.22 .05	.21 .04	.22 .05	.35 .07	1.22 .22	3.95 .10			
-.004	-.005	-.006	3.96 .06	1.08 .22	.31 .08	.20 .05	.19 .04	.20 .05	.29 .07	1.02 .22	3.95 .10	-.005	-.004	-.004
			3.96 .05	.82 .21	.29 .08	.22 .05	.21 .05	.22 .05	2.98 .07	.94 .21	3.95 .11			
			3.96 .07	1.13 .24	.46 .10	.38 .07	.38 .06	.38 .07	.44 .09	1.16 .24	3.95 .13			
			3.96 .08	2.89 .22	1.88 .23	1.68 .19	1.67 .19	1.69 .19	1.86 .22	2.99 .24	3.96 .13			
			3.99 .07	3.97 .11	3.95 .15	3.94 .18	3.94 .20	3.94 .20	3.94 .18	3.96 .14	3.99 .10			
							-.003							
							-.003							
							-.003							

TABLE II.III

GRAVSAT - GEOSTATIONARY

A POSTERIORI DENSITY SIGMAS

21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:

81 DENSITY BLOCKS, σ = 4 MGAL

LOW SATELLITE STATE, σ [HCL]= (1M,2.5M,5M)

HIGH SATELLITE STATE, σ [HCL]= (10M,25M,50M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:

144 DENSITY BLOCKS, σ = 4 MGAL

							-.002							
							-.003							
							-.003							
			3.99 .037	3.98 .060	3.96 .082	3.96 .098	3.95 .108	3.96 .109	3.96 .104	3.97 .084	3.99 .054			
			3.98 .050	3.51 .152	1.89 .245	1.71 .196	1.70 .195	1.70 .192	1.90 .241	3.42 .194	3.97 .073			
			3.97 .053	2.00 .232	.661 .108	.455 .071	.446 .067	.453 .070	.636 .092	1.93 .260	3.96 .073			
			3.97 .048	1.42 .199	.474 .088	.296 .051	.289 .046	.299 .048	.470 .076	1.37 .205	3.96 .066			
-.004	-.005	-.006	3.96 .049	1.19 .201	.414 .084	.274 .046	.267 .041	.276 .044	.410 .074	1.16 .209	3.96 .062	-.006	-.005	-.004
			3.96 .047	.919 .200	.401 .087	.296 .050	.291 .046	.300 .048	.419 .077	1.07 .201	3.96 .064			
			3.96 .053	1.29 .232	.573 .102	.459 .071	.450 .068	.457 .069	.562 .094	1.33 .242	3.96 .073			
			3.97 .057	2.91 .211	1.91 .242	1.72 .195	1.71 .196	1.72 .194	1.91 .237	3.04 .216	3.97 .074			
			3.99 .039	3.97 .064	3.96 .082	3.96 .095	3.96 .104	3.96 .105	3.96 .100	3.97 .081	3.99 .053			
							-.003							
							-.003							
							-.002							

TABLE II.IV GRAVSAT - GEOSTATIONARY

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (2M,5M,10M)
HIGH SATELLITE STATE, σ [HCL]= (20M,50M,100M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							-.002							
							-.002							
							-.0005							
			4.00 / .019	3.98 / .033	3.97 / .045	3.97 / .050	3.97 / .055	3.97 / .057	3.97 / .059	3.98 / .050	3.99 / .030			
			3.98 / .030	3.57 / .113	2.11 / .215	1.93 / .183	1.93 / .186	1.94 / .186	2.14 / .241	3.50 / .158	3.98 / .045			
			3.98 / .031	2.48 / .155	1.18 / .106	1.03 / .065	1.02 / .062	1.05 / .065	1.18 / .103	2.49 / .197	3.97 / .043			
			3.97 / .028	1.80 / .129	.982 / .094	.858 / .049	.851 / .042	.876 / .046	.981 / .088	1.78 / .131	3.97 / .032			
-.004	-.005	-.006	3.97 / .029	1.68 / .125	.948 / .093	.834 / .046	.827 / .039	.850 / .044	.949 / .087	1.69 / .117	3.97 / .025	-.006	-.005	-.004
			3.97 / .029	1.53 / .128	.950 / .093	.842 / .049	.832 / .042	.856 / .047	.963 / .089	1.59 / .124	3.97 / .029			
			3.97 / .031	1.92 / .149	1.11 / .103	.991 / .066	.977 / .062	1.00 / .066	1.12 / .103	2.03 / .180	3.97 / .040			
			3.97 / .036	3.05 / .161	2.11 / .213	1.92 / .182	1.91 / .186	1.93 / .189	2.13 / .236	3.16 / .189	3.97 / .048			
			3.99 / .022	3.98 / .038	3.97 / .045	3.97 / .049	3.97 / .053	3.97 / .056	3.97 / .059	3.97 / .050	3.99 / .030			
							-.0006							
							-.002							
							-.002							

TABLE II.V

GRAVSAT - GEOSTATIONARY

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (10M,25M,50M)
HIGH SATELLITE STATE, σ [HCL]= (100M,250M,500M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							-.0007							
							.0004							
							.004							
			4.00 .013	3.99 .024	3.98 .032	3.97 .036	3.97 .039	3.97 .041	3.97 .043	3.98 .036	4.00 .021			
			3.99 .021	3.62 .088	2.36 .171	2.19 .154	2.18 .157	2.20 .159	2.41 .198	3.56 .124	3.98 .032			
			3.98 .021	2.74 .117	1.50 .071	1.38 .049	1.37 .045	1.40 .048	1.51 .070	2.71 .148	3.98 .030			
			3.98 .021	1.94 .106	1.25 .066	1.17 .040	1.17 .033	1.19 .037	1.26 .062	1.88 .110	3.98 .023			
-.004	-.005	-.006	3.98 .020	1.84 .101	1.22 .067	1.14 .040	1.14 .033	1.16 .037	1.22 .063	1.82 .098	3.98 .019	-.005	-.004	-.003
			3.98 .021	1.72 .101	1.23 .066	1.15 .040	1.14 .033	1.17 .038	1.24 .063	1.72 .104	3.98 .022			
			3.98 .022	2.14 .116	1.43 .070	1.33 .049	1.32 .045	1.34 .050	1.44 .070	2.26 .144	3.98 .030			
			3.98 .027	3.16 .131	2.35 .169	2.16 .153	2.15 .158	2.17 .160	2.38 .192	3.25 .157	3.98 .037			
			4.00 .015	3.98 .028	3.98 .033	3.97 .035	3.97 .038	3.97 .040	3.97 .044	3.98 .038	3.99 .022			
							.004							
							.0003							
							-.0007							

TABLE II.VI

GRAVSAT - GEOSTATIONARY

A POSTERIORI DENSITY SIGMAS

21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:

81 DENSITY BLOCKS, σ = 4 MGAL

LOW SATELLITE STATE, σ [HCL]= (20M,50M,100M)

HIGH SATELLITE STATE, σ [HCL]= (200M,500M,1000M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:

144 DENSITY BLOCKS, σ = 4 MGAL

							.001							
							.004							
							.010							
			4.00 / .01	3.99 / .02	3.98 / .02	3.98 / .02	3.98 / .02	3.98 / .03	3.98 / .03	3.99 / .02	4.00 / .01			
			3.99 / .01	3.70 / .06	2.69 / .13	2.52 / .11	2.51 / .11	2.52 / .11	2.73 / .14	3.65 / .08	3.99 / .02			
			3.99 / .01	3.05 / .07	1.80 / .04	1.68 / .02	1.68 / .02	1.69 / .03	1.82 / .04	2.99 / .08	3.99 / .02			
			3.98 / .02	2.07 / .09	1.47 / .05	1.41 / .03	1.41 / .03	1.42 / .03	1.47 / .45	1.99 / .09	3.98 / .02			
-.003	-.003	-.005	3.98 / .02	1.96 / .09	1.41 / .05	1.36 / .03	1.36 / .03	1.37 / .03	1.42 / .05	1.92 / .09	3.98 / .02	-.004	-.003	-.002
			3.98 / .02	1.83 / .09	1.45 / .05	1.38 / .03	1.39 / .03	1.40 / .03	1.46 / .05	1.81 / .09	3.98 / .02			
			3.98 / .01	2.37 / .08	1.73 / .04	1.64 / .02	1.63 / .02	1.65 / .02	1.75 / .04	2.50 / .09	3.98 / .02			
			3.98 / .02	3.30 / .10	2.67 / .01	2.49 / .11	2.48 / .12	2.50 / .11	2.70 / .13	3.37 / .11	3.98 / .02			
			4.00 / .01	3.99 / .02	3.98 / .02	3.98 / .02	3.98 / .02	3.98 / .02	3.98 / .03	3.99 / .02	4.00 / .01			
							.010							
							.004							
							.001							

TABLE II.VII

GRAVSAT - GEOSTATIONARY

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (40M,100M,200M)
HIGH SATELLITE STATE, σ [HCL]= (400M,1000M,2000M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							-0.02							
							-0.03							
							- .03							
			49.56 / 5.76	48.38 / 8.39	45.50 / 13.28	44.20 / 15.41	43.66 / 15.71	44.55 / 15.78	45.96 / 15.24	48.29 / 10.76	49.46 / 7.70			
			48.95 / 5.27	14.94 / 3.23	6.48 / 1.71	4.98 / 1.61	4.94 / 1.88	5.35 / 2.22	6.60 / 2.48	13.98 / 3.15	48.50 / 8.41			
			47.64 / 4.41	8.16 / 2.50	1.47 / .93	.97 / .66	.88 / .60	.98 / .67	1.29 / .80	7.24 / 1.897	46.55 / 10.25			
			46.38 / 3.46	7.04 / 2.43	1.29 / .87	.78 / .58	.66 / .50	.72 / .53	1.00 / .64	6.13 / 1.58	45.26 / 10.81			
-.05	-.06	-.07	44.33 / 3.52	6.62 / 2.41	1.27 / .88	.75 / .56	.61 / .47	.66 / .49	.92 / .60	5.66 / 1.40	44.02 / 11.63	-.05	-.04	-/04
			46.07 / 3.11	6.64 / 2.34	1.30 / .91	.79 / .58	.66 / .50	.72 / .52	.98 / .64	5.51 / 1.40	44.89 / 12.31			
			44.45 / 4.97	6.86 / 2.37	1.45 / .96	.98 / .66	.89 / .60	.99 / .67	1.24 / .81	5.895 / 1.57	45.31 / 12.10			
			48.15 / 4.82	11.41 / 3.11	5.84 / 1.62	5.03 / 1.58	5.08 / 1.88	5.48 / 2.26	6.18 / 2.34	11.26 / 3.04	47.68 / 9.36			
			49.55 / 5.77	48.44 / 8.45	45.24 / 12.60	44.47 / 14.91	44.19 / 15.41	44.90 / 15.39	45.67 / 14.22	4.82 / 10.64	4.94 / 8.00			
							-.03							
							-.03							
							-.04							

TABLE II.VIII

GRAVSAT - GEOSTATIONARY

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 50 MGAL
LOW SATELLITE STATE, σ [HCL]= (2M,5M,10M)
HIGH SATELLITE STATE, σ [HCL]= (20M,50M,100M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 50 MGAL

							-.002							
							-.003							
							-.002							
			49.23 / .46	47.66 / .67	43.45 / 1.06	41.45 / 1.23	40.96 / 1.21	41.68 / 1.26	43.37 / 1.22	47.08 / .86	48.86 / .62			
			48.67 / .42	14.59 / .26	6.25 / .14	4.72 / .13	4.58 / .15	4.87 / .18	6.12 / .20	13.6 / .25	47.8 / .67			
			47.4 / .35	7.77 / .2	1.14 / .07	.720 / .05	.64 / .05	.72 / .05	1.01 / .06	6.98 / .15	45.42 / .82			
			46.26 / .28	6.60 / .19	.95 / .07	.53 / .05	.43 / .04	.49 / .04	.77 / .05	5.9 / .13	43.96 / .87			
-.004	-.005	-.006	44.19 / .28	6.17 / .19	.926 / .07	.49 / .04	.39 / .04	.44 / .04	.70 / .05	5.49 / .11	42.46 / .93	-.004	-.003	-.003
			45.96 / .25	6.21 / .19	.93 / .07	.53 / .05	.43 / .04	.49 / .04	.74 / .05	5.33 / .11	43.18 / .99			
			44.18 / .4	6.43 / .19	1.09 / .08	.73 / .05	.66 / .05	.73 / .05	.95 / .06	5.68 / .13	43.67 / .97			
			47.91 / .39	10.98 / .25	5.61 / .13	4.78 / .13	4.72 / .15	4.99 / .18	5.72 / .19	10.84 / .24	46.76 / .75			
			49.21 / .46	47.70 / .68	43.46 / 1.01	41.92 / 1.19	41.43 / 1.23	42.20 / 1.23	43.41 / 1.14	47.05 / .85	48.77 / .64			
							-.002							
							-.003							
							-.002							

TABLE II.IX GRAVSAT - GEOSTATIONARY

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 50 MGAL
LOW SATELLITE STATE, σ [HCL]= (2M,5M,10M)
HIGH SATELLITE STATE, σ [HCL]= (20M,50M,100M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							-.002							
							-.003							
							-.003							
			3.99 .14	3.98 .24	3.98 .38	3.97 .38	3.96 .34	3.96 .32	3.96 .29	3.97 .22	3.99 .13			
			3.98 .15	3.52 .30	1.91 .37	1.72 .36	1.72 .37	1.73 .39	1.92 .40	3.44 .47	3.97 .16			
			3.97 .16	2.07 .73	.71 .34	.48 .23	.47 .19	.47 .19	.66 .26	1.96 .56	3.96 .17			
			3.97 .13	1.46 .52	.52 .29	.32 .17	.31 .14	.31 .14	.49 .20	1.39 .41	3.96 .17			
-.004	-.005	-.006	3.96 .13	1.25 .55	.46 .28	.30 .17	.29 .14	.29 .14	.43 .20	1.17 .34	3.96 .16	-.005	-.004	-.004
			3.97 .12	.99 .51	.45 .28	.32 .17	.31 .14	.31 .14	.43 .19	1.07 .32	3.96 .17			
			3.96 .15	1.36 .64	.62 .33	.49 .23	.47 .19	.47 .18	.58 .24	1.34 .38	3.96 .18			
			3.97 .15	2.92 .35	1.93 .38	1.74 .35	1.73 .36	1.74 .38	1.93 .44	3.08 .69	3.97 .17			
			3.99 .14	3.98 .25	3.98 .39	3.97 .38	3.97 .34	3.96 .31	3.96 .28	3.97 .23	3.99 .14			
							-.003							
							-.003							
							-.003							

TABLE II.X GRAVSAT - GEOSTATIONARY

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (2M,5M,10M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL
HIGH SATELLITE STATE, σ [HCL]= (20M,50M,100M)

							-.002							
							-.003							
							-.003							
			3.99 / .079	3.95 / .13	3.93 / .19	3.90 / .29	3.89 / .35	3.89 / .35	3.91 / .31	3.94 / .22	3.98 / .15			
			3.96 / .08	2.06 / .27	.95 / .21	.71 / .18	.70 / .17	.71 / .17	.92 / .20	1.95 / .34	3.94 / .17			
			3.94 / .63	1.05 / .23	.37 / .09	.27 / .07	.26 / .06	.28 / .07	.38 / .08	1.13 / .27	3.93 / .15			
			3.92 / .05	.81 / .21	.33 / .08	.23 / .05	.23 / .04	.24 / .05	.35 / .07	.90 / .23	3.91 / .10			
-.004	-.005	-.006	3.88 / .07	.78 / .20	.32 / .08	.22 / .05	.22 / .04	.23 / .04	.34 / .07	.86 / .21	3.90 / .08	-.005	-.004	-.004
			3.91 / .06	.74 / .20	.32 / .08	.23 / .05	.22 / .05	.23 / .05	.35 / .08	.84 / .20	3.91 / .09			
			3.89 / .08	.81 / .21	.35 / .10	.26 / .07	.25 / .06	.26 / .06	.38 / .10	.90 / .21	3.92 / .13			
			3.94 / .08	1.36 / .29	.82 / .21	.70 / .18	.69 / .18	.70 / .18	.85 / .22	1.48 / .31	3.93 / .15			
			3.99 / .08	3.96 / .12	3.93 / .17	3.91 / .26	3.90 / .32	3.90 / .33	3.91 / .28	3.95 / .20	3.98 / .14			
							-.003							
							-.003							
							-.003							

TABLE X.XI

GRAVSAT - GEOSTATIONARY

A POSTERIORI DENSITY SIGMAS

21 SHORT DATA ARCS (DATA NOISE = .01 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (2M,5M,10M)
HIGH SATELLITE STATE, σ [HCL]= (20M,50M,100M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							-.002							
							-.002							
							-.0005							
			4.00 .02	3.98 .03	3.97 .05	3.97 .05	3.97 .06	3.97 .06	3.97 .06	3.98 .05	3.99 .03			
			3.98 .03	3.57 .11	2.11 .22	1.94 .18	1.93 .19	1.94 .19	2.14 .24	3.50 .16	3.98 .05			
			3.98 .03	2.48 .15	1.18 .11	1.03 .07	1.02 .06	1.05 .06	1.18 .10	2.49 .20	3.97 .04			
			3.97 .03	1.80 .13	.98 .09	.86 .05	.85 .04	.88 .05	.98 .09	1.78 .13	3.97 .03			
-.004	-.005	-.006	3.97 .03	1.68 .12	.95 .09	.84 .05	.83 .04	.85 .04	.95 .09	1.69 .12	3.97 .03	-.006	-.005	-.004
			3.97 .03	1.53 .13	.95 .09	.84 .05	.83 .04	.86 .05	.97 .09	1.59 .12	3.97 .03			
			3.97 .03	1.92 .15	1.12 .10	1.00 .07	.98 .06	1.00 .07	1.13 .10	2.03 .18	3.97 .04			
			3.97 .04	3.05 .16	2.11 .21	1.98 .18	1.92 .19	1.93 .19	2.13 .24	3.16 .19	3.97 .05			
			3.99 .02	3.98 .04	3.97 .05	3.97 .05	3.97 .05	3.97 .06	3.97 .06	3.98 .05	3.99 .03			
							-.0006							
							-.002							
							-.002							

TABLE II.XII GRAVSAT - GEOSTATIONARY (Drag & Drag-Rate Unadjusted)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (10M,25M,50M)
HIGH SATELLITE STATE, σ [HCL]= (100M,250M,500M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

			-.03	-.03	-.02	-.01	-.01	-.003	-.02	-.02	-.02			
			-.03	-.01	.05	.07	-.13	.08	.03	.01	-.02			
			-.03	.06	.08	.01	.41	-.005	.08	.05	-.02			
			-.04	.08	.05	.28	-.56	.28	.04	.07	-.03			
			-.04	.04	.19	-.18	1	.19	.18	.02	-.03			
			-.04	.12	.06	.29	-.57	.29	.03	.10	-.03			
			-.03	.07	.10	.004	.41	-.13	.09	.06	-.03			
			-.03	.02	.04	.07	-.13	.08	.03	.03	-.02			
			-.02	-.02	.02	-.01	.01	-.003	-.02	-.02	-.02			

TABLE II.XIII GRAVSAT - GEOSTATIONARY

CORRELATIONS OF ADJUSTED BLOCKS WITH CENTER BLOCK
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)
HIGH SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

			-.02	-.02	-.02	-.01	.001	-.01	-.01	-.01	-.01			
			-.02	-.01	.31	-.002	.003	-.01	.03	-.01	-.02			
			-.02	-.02	.12	-.22	.60	-.23	.12	-.03	-.02			
			-.03	-.004	.12	-.22	.41	-.22	.12	-.02	-.02			
			-.03	-.04	.18	-.40	1	-.40	.17	-.04	-.02			
			-.02	.02	.13	-.21	.40	-.21	.12	-.006	-.02			
			-.02	-.01	.12	-.21	.59	-.22	.13	-.01	-.02			
			-.02	.004	.03	.003	-.005	-.001	.03	-.006	-.02			
			-.02	-.02	-.02	-.01	.001	-.007	-.01	-.02	-.02			

TABLE II.XIV

GRAVSAT - GEOSTATIONARY

CORRELATIONS OF ADJUSTED BLOCKS WITH CENTER BLOCK
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (2M,5M,10M)
HIGH SATELLITE STATE, σ [HCL]= (20M,50M,100M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

SET III:

Solution Set III investigates the "high-high-low" GRAVSAT/GEOSTATIONARY configuration utilizing two high spacecraft described in Section 6.1. The two high satellites, 60° to the east and west of the center block, simultaneously observe the low satellite throughout all twenty-one arcs, introducing more cross track geometry in the data type and resulting in twice the data coverage of the single high geostationary satellite of solution Set II. Thus, in comparing with Sets I and II, the error analysis results in Tables III.I-III.VI should be multiplied by $\sqrt{2}$.

Tables III.I-III.VI repeat the studies of Tables II.II-II.VII for a single high spacecraft over the center of the data region. Considering the $\sqrt{2}$ factor, we find the same result observed when comparing the coplanar and planar offset GRAVSAT/GEOPAUSE configurations of Set I; for large values of the a priori uncertainty of unadjusted density blocks (50 mgal) the "out-of-plane" geometry of the two high satellites (or planar offset GEOPAUSE) provides a better recovery, while for a smaller more reasonable value of unadjusted density block a priori for fine structure estimation (4 mgal) the more "in plane" configuration provides the better recovery. This is due to the fact that for short arcs, although the low satellite is most strongly perturbed by the unadjusted blocks nearest the ground track, large uncertainties in unadjusted blocks away from the ground track represent what can be substantial "out-of-plane" orbital perturbations, while for small uncertainties of the unadjusted blocks, the orbital perturbations are almost totally planar.

To illustrate this effect, Tables III.VII-III.XII repeat the studies of Tables III.I-III.VI, but with only the single high satellite 60° to the west of the center block observing the GRAVSAT. The total data used is then half that of Tables III.I-III.VI, and exactly the same as the "high-low" studies of Sets I and II. Comparing with the "high-low" GRAVSAT/GEOSTATIONARY case of Set II (with 4 mgal a priori uncertainties on the density blocks and the high satellite directly over the center block) we find that, for the smaller epoch state errors, the recovery uncertainties of Tables III.VII-III.X are nearly twice those of Set II. The recovery uncertainties for the large epoch state errors are similar in magnitude. Multiplying the uncertainties of Tables III.I-III.VI by $\sqrt{2}$ and comparing with the uncertainties of Tables III.VII-III.XII shows that the added geometry of the second high satellite 60° east of center improves the solution uncertainty of the single high satellite 60° west of center by approximately 20%.

							-.026							
							-.032							
							-.036							
			91.60 / 81.20	43.67 / 25.02	33.97 / 23.04	34.32 / 29.33	27.96 / 22.78	32.49 / 27.72	31.56 / 20.21	40.94 / 22.70	97.24 / 87.78			
			47.58 / 29.87	14.03 / 10.65	3.74 / 2.35	4.61 / 4.09	4.51 / 4.06	4.64 / 4.17	4.24 / 3.18	15.96 / 13.87	49.21 / 34.33			
			39.28 / 25.98	42.29 / 2.41	.69 / .51	.54 / .40	.54 / .41	.54 / .40	.65 / .46	3.92 / 2.50	36.66 / 23.84			
			46.31 / 39.78	4.18 / 3.60	.49 / .41	.35 / .28	.37 / .31	.35 / .29	.42 / .32	2.57 / 1.63	35.48 / 26.93			
-.016	-.014	-.010	26.14 / 16.88	2.64 / 2.11	.48 / .42	.29 / .24	.28 / .22	.28 / .22	.48 / .43	2.59 / 2.04	29.99 / 22.01	.0003	-.005	-.009
			26.99 / 21.09	1.23 / .55	.38 / .29	.33 / .26	.35 / .29	.34 / .28	.41 / .32	1.50 / .53	28.37 / 21.58			
			25.52 / 15.38	2.31 / 1.58	.66 / .50	.55 / .42	.54 / .41	.53 / .40	.67 / .52	2.26 / 1.51	24.53 / 14.37			
			39.29 / 25.44	14.74 / 13.26	4.54 / 3.60	4.46 / 3.93	4.67 / 4.25	4.74 / 4.30	4.65 / 3.86	13.00 / 11.79	36.96 / 23.21			
			122.25 / 115.89	43.65 / 29.26	37.08 / 28.78	32.28 / 26.96	29.05 / 23.97	31.29 / 26.29	32.94 / 24.99	41.82 / 27.40	114.39 / 107.54			
							-.036							
							-.032							
							-.026							

TABLE III.1 GRAVSAT - 2 GEOSTATIONARY

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 50 MGAL
LOW SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)
HIGH SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 50 MGAL

							-.002							
							-.003							
							-.003							
			3.94 / .48	3.88 / .57	3.85 / .57	3.83 / .43	3.82 / .33	3.83 / .42	3.85 / .56	3.88 / .58	3.94 / .48			
			3.86 / .66	2.97 / .63	1.80 / .30	1.59 / .26	1.57 / .25	1.59 / .25	1.76 / .28	2.92 / .67	3.85 / .66			
			3.79 / .79	1.50 / .33	.49 / .08	.43 / .07	.44 / .07	.44 / .07	.49 / .08	1.46 / .35	3.76 / .79			
			3.72 / .90	1.06 / .26	.25 / .05	.21 / .04	.21 / .03	.22 / .04	.25 / .05	1.11 / .33	3.69 / .83			
-.0004	.0007	.003	3.51 / 1.05	.94 / .21	.22 / .03	.20 / .03	.20 / .03	.20 / .03	.23 / .03	1.05 / .30	3.52 / .87	.004	.001	-.00009
			3.42 / 1.21	.84 / .15	.24 / .05	.21 / .04	.21 / .03	.21 / .04	.25 / .05	.98 / .27	3.44 / .98			
			3.48 / 1.12	1.13 / .21	.46 / .07	.42 / .07	.43 / .07	.43 / .07	.48 / .07	1.25 / .27	3.41 / 1.04			
			3.78 / .83	2.62 / .60	1.64 / .26	1.56 / .25	1.55 / .25	1.56 / .25	1.72 / .27	2.75 / .67	3.77 / .80			
			3.93 / .56	3.87 / .62	3.84 / .58	3.82 / .43	3.82 / .35	3.82 / .44	3.84 / .58	3.87 / .62	3.93 / .54			
							-.003							
							-.003							
							-.002							

TABLE III.11

GRAVSAT - 2 GEOSTATIONARY

A POSTERIORI DENSITY SIGMAS

21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (1M,2.5M,5M)
HIGH SATELLITE STATE, σ [HCL]= (10M,25M,50M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							-.003							
							-.003							
							-.003							
			3.94 / .36	3.89 / .46	3.86 / .47	3.86 / .34	3.85 / .23	3.86 / .32	3.86 / .45	3.89 / .46	3.94 / .36			
			3.86 / .50	3.00 / .62	1.84 / .31	1.63 / .26	1.61 / .26	1.62 / .25	1.80 / .29	2.95 / .65	3.86 / .51			
			3.82 / .55	1.63 / .44	.53 / .08	.47 / .07	.47 / .07	.47 / .07	.53 / .08	1.62 / .47	3.80 / .54			
			3.75 / .58	1.16 / .36	.27 / .05	.24 / .04	.24 / .04	.25 / .04	.29 / .06	1.21 / .42	3.73 / .54			
.0006	.002	.005	3.51 / .75	1.05 / .32	.25 / .04	.22 / .03	.23 / .03	.23 / .03	.27 / .04	1.18 / .41	3.54 / .62	.005	.002	.0005
			3.47 / .85	.99 / .28	.26 / .05	.23 / .04	.24 / .04	.24 / .04	.28 / .05	1.12 / .37	3.52 / .69			
			3.53 / .83	1.30 / .32	.49 / .07	.46 / .07	.46 / .07	.46 / .07	.52 / .07	1.46 / .39	3.46 / .83			
			3.78 / .66	2.65 / .61	1.68 / .27	1.60 / .26	1.59 / .26	1.60 / .25	1.77 / .28	2.80 / .63	3.77 / .64			
			3.93 / .43	3.88 / .50	3.86 / .47	3.85 / .33	3.85 / .26	3.85 / .34	3.86 / .46	3.88 / .48	3.94 / .41			
							-.003							
							-.003							
							-.003							

TABLE III.III

GRAVSAT - 2 GEOSTATIONARY

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (2M,5M,10M)
HIGH SATELLITE STATE, σ [HCL]= (20M,50M,100M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							.0009							
							.0005							
							.002							
			3.96 .20	3.91 .29	3.88 .20	3.88 .23	3.88 .16	3.89 .22	3.89 .29	3.91 .27	3.96 .19			
			3.90 .27	3.11 .48	1.98 .27	1.76 .24	1.75 .24	1.78 .23	1.97 .25	3.08 .47	3.89 .26			
			3.88 .25	1.99 .43	.78 .08	.72 .07	.74 .07	.77 .08	.85 .10	2.02 .44	3.87 .23			
			3.80 .31	1.43 .32	.56 .04	.53 .04	.55 .04	.58 .04	.65 .05	1.49 .32	3.79 .30			
.004	.007	.013	3.61 .46	1.38 .30	.53 .04	.51 .03	.53 .04	.56 .04	.62 .06	1.52 .33	3.60 .46	.012	.006	.003
			3.69 .35	1.41 .32	.54 .04	.52 .04	.54 .04	.58 .04	.63 .05	1.48 .32	3.71 .31			
			3.70 .40	1.71 .36	.74 .08	.71 .07	.73 .07	.76 .08	.83 .09	1.91 .41	3.67 .41			
			3.83 .37	2.83 .48	1.82 .24	1.74 .24	1.74 .24	1.76 .23	1.94 .24	2.98 .44	3.82 .36			
			3.95 .24	3.90 .31	3.88 .30	3.88 .23	3.88 .18	3.88 .22	3.89 .28	3.91 .28	3.95 .22			
							.002							
							.0004							
							.0009							

TABLE III.IV GRAVSAT - 2 GEOSTATIONARY

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (10M,25M,50M)
HIGH SATELLITE STATE, σ [HCL]= (100M,250M,500M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							.002							
							.004							
							.013							
			3.97 / .15	3.92 / .23	3.90 / .24	3.90 / .19	3.90 / .13	3.90 / .17	3.90 / .23	3.93 / .22	3.97 / .15			
			3.92 / .20	3.20 / .40	2.13 / .23	1.93 / .21	1.94 / .21	1.96 / .21	2.14 / .21	3.17 / .40	3.92 / .20			
			3.90 / .17	2.20 / .37	1.06 / .09	1.00 / .07	1.02 / .07	1.06 / .08	1.15 / .11	2.21 / .37	3.90 / .16			
			3.83 / .26	1.57 / .28	.83 / .05	.81 / .04	.84 / .04	.88 / .06	.94 / .07	1.60 / .28	3.81 / .26			
.004	.008	.015	3.66 / .39	1.55 / .27	.81 / .06	.79 / .05	.82 / .05	.86 / .06	.92 / .08	1.63 / .28	3.65 / .41	.013	.007	.003
			3.74 / .28	1.55 / .28	.82 / .05	.80 / .04	.83 / .04	.87 / .05	.93 / .07	1.59 / .28	3.74 / .27			
			3.77 / .28	1.91 / .32	1.02 / .08	1.00 / .07	1.02 / .07	1.06 / .08	1.14 / .10	2.09 / .36	3.74 / .30			
			3.86 / .28	2.94 / .40	1.99 / .21	1.91 / .21	1.92 / .21	1.95 / .20	2.12 / .21	3.07 / .37	3.86 / .28			
			3.96 / .18	3.92 / .24	3.90 / .24	3.90 / .19	3.90 / .15	3.90 / .18	3.90 / .23	3.92 / .22	3.96 / .17			
							.014							
							.004							
							.002							

TABLE III.V

GRAVSAT - 2 GEOSTATIONARY

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (20M,50M,100M)
HIGH SATELLITE STATE, σ [HCL]= (200M,500M,1000M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							.004							
							.009							
							.022							
			3.97 / .11	3.94 / .17	3.92 / .18	3.92 / .14	3.92 / .10	3.92 / .13	3.92 / .18	3.94 / .17	3.97 / .11			
			3.94 / .15	3.28 / .34	2.31 / .19	2.14 / .19	2.15 / .18	2.17 / .19	2.32 / .19	3.26 / .34	3.93 / .16			
			3.92 / .13	2.38 / .32	1.36 / .09	1.31 / .08	1.33 / .07	1.35 / .08	1.43 / .11	2.38 / .31	3.92 / .12			
			3.86 / .20	1.66 / .26	1.12 / .08	1.10 / .06	1.12 / .05	1.15 / .07	1.20 / .09	1.67 / .26	3.83 / .21			
.003	.006	.012	3.75 / .29	1.65 / .26	1.11 / .08	1.08 / .07	1.11 / .06	1.13 / .08	1.18 / .10	1.69 / .26	3.73 / .31	.012	.005	.003
			3.78 / .22	1.63 / .26	1.13 / .07	1.10 / .06	1.12 / .06	1.15 / .07	1.20 / .09	1.65 / .26	3.76 / .23			
			3.83 / .19	2.11 / .29	1.35 / .09	1.32 / .07	1.34 / .07	1.37 / .08	1.44 / .10	2.26 / .30	3.81 / .20			
			3.90 / .20	3.04 / .35	2.22 / .18	2.14 / .18	2.14 / .19	2.17 / .18	2.32 / .18	3.16 / .31	3.89 / .21			
			3.97 / .13	3.93 / .18	3.92 / .18	3.92 / .14	3.92 / .11	3.92 / .14	3.92 / .18	3.94 / .17	3.97 / .13			
							.022							
							.009							
							.004							

TABLE III.VI GRAVSAT - 2 GEOSTATIONARY

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (40M,100M,200M)
HIGH SATELLITE STATE, σ [HCL]= (400M,1000M,2000M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							-.004							
							-.004							
							-.005							
			3.98/.36	3.96/.44	3.88/.59	3.79/.81	3.76/.98	3.79/1.06	3.83/1.04	3.87/.80	3.95/.57			
			3.96/.47	3.76/.33	1.75/.20	1.50/.24	1.53/.25	1.60/.25	1.83/.28	2.64/.65	3.88/.62			
			3.88/.66	3.21/.45	.70/.13	.50/.10	.51/.11	.55/.12	.64/.13	1.40/.42	3.86/.61			
			3.79/.91	3.14/.42	.58/.098	.35/.07	.30/.06	.30/.06	.36/.08	1.12/.37	3.83/.57			
.003	.007	.02	3.75/1.15	3.09/.22	.55/.08	.35/.06	.28/.05	.28/.05	.33/.08	1.05/.35	3.78/.60	-.004	-.004	-.004
			3.70/1.31	2.35/.20	.41/.09	.32/.07	.28/.06	.30/.06	.34/.08	1.03/.33	3.69/.71			
			3.76/1.06	2.52/.22	.55/.12	.47/.10	.49/.10	.53/.11	.61/.12	1.31/.38	3.70/.78			
			3.94/.56	3.63/.28	1.48/.23	1.48/.25	1.49/.26	1.57/.26	1.71/.27	2.36/.59	3.83/.72			
			3.98/.31	3.96/.31	3.86/.49	3.78/.80	3.77/.98	3.79/1.05	3.82/1.03	3.86/.83	3.94/.61			
							-.002							
							-.003							
							-.003							

TABLE III.VII GRAVSAT - 1 GEOSTATIONARY (60° West of Center)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)
HIGH SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							-.003							
							-.004							
							-.005							
			3.99 / .17	3.98 / .18	3.96 / .16	3.93 / .14	3.93 / .12	3.93 / .13	3.93 / .17	3.94 / .21	3.97 / .18			
			3.96 / .27	3.86 / .33	2.32 / .17	2.00 / .24	2.02 / .25	2.08 / .26	2.36 / .34	3.01 / .55	3.91 / .28			
			3.93 / .37	3.65 / .51	.84 / .12	.69 / .09	.69 / .10	.73 / .11	.82 / .13	1.76 / .60	3.88 / .32			
			3.90 / .46	3.40 / .51	.62 / .08	.42 / .06	.36 / .05	.36 / .05	.41 / .06	1.26 / .47	3.88 / .32			
.005	.01	.023	3.83 / .59	3.25 / .39	.59 / .08	.40 / .06	.33 / .05	.34 / .05	.37 / .06	1.24 / .48	3.85 / .37	-.002	-.002	-.003
			3.83 / .61	2.66 / .13	.49 / .11	.38 / .07	.34 / .06	.35 / .06	3.8 / .07	1.22 / .46	3.81 / .42			
			3.87 / .47	2.87 / .26	.69 / .15	.64 / .11	.66 / .12	.71 / .13	.79 / .13	1.72 / .61	3.78 / .44			
			3.95 / .30	3.71 / .25	2.02 / .24	1.98 / .25	1.98 / .27	2.05 / .27	2.26 / .33	2.86 / .57	3.87 / .35			
			3.99 / .18	3.98 / .17	3.95 / .14	3.93 / .13	3.93 / .12	3.93 / .14	3.93 / .18	3.93 / .23	3.97 / .20			
							-.005							
							-.004							
							-.003							

TABLE III.VIII GRAVSAT - 1 GEOSTATIONARY (60° West of Center)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (1M,2.5M,5M)
HIGH SATELLITE STATE, σ [HCL]= (10M,25M,50M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							-.003							
							-.004							
							-.005							
			3.99 .11	3.99 .12	3.97 .10	3.94 .09	3.94 .09	3.94 .10	3.94 .14	3.94 .19	3.97 .15			
			3.97 .18	3.88 .25	2.38 .18	2.05 .24	2.08 .25	2.13 .26	2.41 .38	3.05 .54	3.91 .25			
			3.95 .25	3.73 .38	.91 .12	.76 .09	.17 .10	.79 .11	.88 .13	1.84 .62	3.89 .29			
			3.92 .30	3.56 .37	.68 .08	.49 .06	.43 .05	.42 .06	.47 .07	1.32 .48	3.88 .28			
.007	.01	.03	3.88 .36	3.31 .31	.64 .08	.47 .06	.41 .06	.40 .06	.43 .06	1.31 .50	3.86 .31	-.0006	-.002	-.002
			3.87 .36	2.82 .14	.56 .11	.46 .07	.41 .06	.41 .06	.43 .07	1.31 .49	3.84 .34			
			3.90 .29	3.10 .27	.75 .14	.72 .11	.72 .11	.77 .12	.84 .13	1.82 .64	3.82 .36			
			3.96 .20	3.75 .21	2.08 .23	2.03 .24	2.04 .26	2.10 .27	2.32 .33	2.93 .56	3.88 .30			
			3.99 .12	3.98 .12	3.96 .10	3.94 .09	3.94 .09	3.94 .11	3.94 .15	3.94 .19	3.97 .17			
								-.005						
								-.004						
								-.003						

TABLE III.IX GRAVSAT - 1 GEOSTATIONARY (60° West of Center)

A POSTERIORI DENSITY SIGMAS

21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (2M,5M,10M)
HIGH SATELLITE STATE, σ [HCL]= (20M,50M,100M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							-.0005							
							.0003							
							.0004							
			4.00 / .04	3.99 / .04	3.98 / .04	3.96 / .05	3.95 / .06	3.95 / .07	3.95 / .10	3.95 / .12	3.98 / .09			
			3.98 / .06	3.92 / .11	2.59 / .15	2.29 / .20	2.31 / .21	2.35 / .23	2.59 / .29	3.17 / .40	3.93 / .15			
			3.98 / .07	3.85 / .14	1.44 / .09	1.28 / .08	1.28 / .08	1.29 / .08	1.36 / .09	2.22 / .40	3.92 / .16			
			3.97 / .08	3.71 / .11	1.18 / .07	1.03 / .06	.991 / .05	.96 / .05	.98 / .06	1.57 / .31	3.91 / .14			
.007	.01	.03	3.94 / .08	3.55 / .11	1.15 / .07	1.02 / .06	.99 / .05	.96 / .04	.96 / .05	1.57 / .29	3.90 / .14	.0008	-.0004	-.0009
			3.95 / .08	3.40 / .11	1.14 / .07	1.03 / .06	.99 / .05	.95 / .05	.95 / .05	1.55 / .31	3.90 / .15			
			3.94 / .10	3.37 / .13	1.35 / .10	1.29 / .08	1.28 / .08	1.28 / .08	1.32 / .09	2.15 / .42	3.88 / .18			
			3.98 / .07	3.80 / .09	2.37 / .18	2.28 / .20	2.88 / .22	2.32 / .23	2.51 / .27	3.08 / .42	3.91 / .18			
			4.00 / .04	3.99 / .04	3.97 / .05	3.96 / .06	3.95 / .07	3.95 / .08	3.95 / .10	3.95 / .12	3.98 / .10			
							.004							
							.0005							
							-.0004							

TABLE III.X

GRAVSAT - 1 GEOSTATIONARY (60° West of Center)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (10M,25M,50M)
HIGH SATELLITE STATE, σ [HCL]= (100M,250M,500M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							.002							
							.005							
							.01							
			4.00 / .02	3.99 / .03	3.98 / .03	3.96 / .04	3.96 / .05	3.96 / .06	3.96 / .07	3.96 / .09	3.98 / .07			
			3.99 / .04	3.93 / .07	2.72 / .12	2.48 / .16	2.48 / .17	2.52 / .18	2.73 / .23	3.26 / .32	3.94 / .12			
			3.99 / .04	3.88 / .08	1.70 / .07	1.55 / .05	1.56 / .05	1.58 / .05	1.65 / .05	2.39 / .31	3.93 / .11			
			3.98 / .04	3.75 / .06	1.41 / .07	1.28 / .05	1.25 / .05	1.23 / .05	1.25 / .06	1.66 / .25	3.92 / .10			
.005	.01	.02	3.95 / .04	3.62 / .06	1.39 / .07	1.28 / .05	1.25 / .05	1.24 / .05	1.25 / .06	1.68 / .23	3.91 / .09	-.0003	-.0007	-.0007
			3.96 / .04	3.56 / .06	1.39 / .07	1.29 / .05	1.25 / .05	1.23 / .05	1.24 / .06	1.65 / .25	3.91 / .10			
			3.96 / .06	3.47 / .09	1.63 / .07	1.58 / .05	1.57 / .05	1.58 / .05	1.63 / .05	2.33 / .31	3.90 / .13			
			3.98 / .04	3.82 / .06	2.56 / .15	2.47 / .16	2.47 / .18	2.50 / .18	2.68 / .22	3.18 / .34	3.92 / .13			
			4.00 / .02	3.99 / .03	3.97 / .03	3.96 / .04	3.96 / .05	3.95 / .06	3.96 / .07	3.96 / .09	3.98 / .07			
						.01	.01							
						.003	.005							
						.001	.002							

TABLE III.XI GRAVSAT - 1 GEOSTATIONARY (60° West of Center)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (20M,50M,100M)
HIGH SATELLITE STATE, σ [HCL]= (200M,500M,1000M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							.004							
							.008							
							.02							
			4.00 / .01	4.00 / .01	3.98 / .02	3.97 / .03	3.97 / .04	3.96 / .04	3.96 / .05	3.97 / .06	3.99 / .04			
			3.99 / .02	3.95 / .04	2.85 / .11	2.65 / .13	2.64 / .14	2.66 / .15	2.86 / .17	3.36 / .24	3.96 / .08			
			3.99 / .02	3.91 / .04	1.88 / .06	1.74 / .04	1.74 / .04	1.76 / .04	1.83 / .04	2.35 / .23	3.95 / .08			
			3.98 / .02	3.80 / .05	1.54 / .06	1.43 / .06	1.40 / .06	1.38 / .06	1.39 / .07	1.71 / .23	3.93 / .08			
.003	.007	.002	3.97 / .03	3.69 / .04	1.52 / .07	1.42 / .06	1.39 / .06	1.38 / .06	1.41 / .07	1.72 / .22	3.92 / .08	-.0009	-.0009	-.0007
			3.97 / .03	3.63 / .04	1.59 / .07	1.43 / .06	1.40 / .06	1.38 / .06	1.40 / .07	1.70 / .23	3.92 / .08			
			3.98 / .03	3.63 / .05	1.82 / .06	1.76 / .04	1.75 / .04	1.76 / .03	1.83 / .04	2.46 / .23	3.92 / .09			
			3.99 / .02	3.87 / .04	2.74 / .13	2.65 / .13	2.64 / .14	2.66 / .14	2.82 / .16	3.30 / .26	3.94 / .09			
			4.00 / .01	3.99 / .02	3.98 / .03	3.97 / .03	3.97 / .04	3.96 / .04	3.96 / .05	3.97 / .06	3.99 / .05			
							.02							
							.007							
							.003							

TABLE III.XII GRAVSAT - 1 GEOSTATIONARY (60°West of Center)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (40M,100M,200M)
HIGH SATELLITE STATE, σ [HCL]= (400M,1000M,2000M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

SET IV:

Solution Set IV investigates the recovery of 2 1/2° x 2 1/2° density blocks with the "high-low" GRAVSAT/GEOSTATIONARY configuration described in Section 6.1. Epoch state a priori uncertainties comparable to those of Tables II.I-II.VII are displayed in Tables IV.I-IV.V with a priori density block uncertainties of 4 mgals and 8 mgals.

Table IV.I presents the most favorable case with virtually no error in the satellite epoch states and a 4 mgal a priori for the density block. Comparing with Table II.I shows a much poorer recovery ability for the 2 1/2° x 2 1/2° resolution of the gravity fine structure. This result is more clearly demonstrated in Tables IV.II-IV.VI. Note that an assumed a priori density block uncertainty of 8 mgal and very accurate epoch state vectors (Table IV.V) results in a recovery error in the center adjusted block of 2.36 mgal, while halving the density block uncertainty to 4 mgal (Table IV.II) results in reducing the error to 1.77 mgal. Tables IV.VII and IV.VIII reproduce the error analysis study of Table IV.III (data noise .05 cm/sec) with the data noise assumed to be .01 cm/sec and .005 cm/sec, respectively. This reduces the recovered uncertainty of 1.86 mgal to .89 mgal and .74 mgal, respectively. Table IV.IX displays the correlations of the adjusted blocks with the center block for the recovery depicted in Table IV.I. Relative to the correlations observed for 5° x 5° blocks of Sets I and II, the nearest neighbors in the north-south direction display a marked increase. However, these correlations indicate no difficulty in resolving blocks of 2 1/2° x 2 1/2° with the GRAVSAT at 250 km. altitude providing highly accurate SST data is available. Satellite-to-satellite tracking for the "high-low" GRAVSAT/GEOSTATIONARY configuration (with the GRAVSAT at 250 km. altitude) does not significantly improve the gravitational fine structure at 2 1/2° x 2 1/2° resolution with data noise at .05 cm/sec. Until state-of-the-art instrumentation is available which will produce SST range-rate data at the .005 cm/sec noise level recovery of such resolution is not feasible.

							.004							
							.005							
							-.003							
			4.00/.29	3.96/1.06	3.93/2.51	3.87/2.49	3.87/2.50	3.87/2.49	3.97/2.56	3.97/1.29	3.99/.54			
			3.99/.17	3.89/.62	3.60/1.32	3.50/1.23	3.50/1.24	3.51/1.22	3.60/1.30	3.88/.65	3.98/.36			
			3.97/.07	3.53/.20	2.30/.69	2.06/.63	2.06/.63	2.07/.62	2.39/.71	3.54/.34	3.96/.22			
			3.95/.08	3.13/.26	1.92/.16	1.72/.14	1.72/.13	1.74/.15	1.96/.21	3.22/.39	3.94/.18			
-.003	-.004	-.005	3.95/.09	2.98/.28	1.87/.12	1.72/.09	1.71/.08	1.73/.11	1.92/.20	3.06/.40	3.93/.15	-.007	-.005	-.004
			3.94/.08	2.90/.27	1.96/.11	1.83/.12	1.81/.13	1.84/.15	2.03/.18	2.99/.41	3.93/.12			
			3.95/.07	3.05/.34	2.48/.50	2.29/.64	2.27/.63	2.30/.62	2.51/.72	3.14/.36	3.94/.18			
			3.98/.06	3.79/.16	3.59/1.06	3.59/1.18	3.55/1.19	3.56/1.20	3.66/1.20	3.78/1.17	3.97/.49			
			3.99/.151	3.98/.52	3.87/2.00	4.01/2.50	3.92/2.46	3.93/2.46	4.01/2.41	3.90/1.99	3.99/.78			
							.00003							
							.007							
							.007							

TABLE IV.I GRAVSAT - GEOSTATIONARY (2 1/2° x 2 1/2° Blocks)

A POSTERIORI DENSITY SIGMAS
10 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PAREMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL] = (.001M,.001M,.001M)
HIGH SATELLITE STATE, σ [HCL] = (.001M,.001M,.001M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							.00003							
							-.003							
							-.012							
			4.00 / .06	3.97 / .18	3.88 / .47	3.87 / .58	3.87 / .62	3.87 / .64	3.87 / .58	3.96 / .27	3.99 / .12			
			3.99 / .04	3.90 / .12	3.59 / .31	3.58 / .33	3.58 / .34	3.58 / .36	3.60 / .35	3.88 / .19	3.98 / .09			
			3.98 / .04	3.62 / .12	2.40 / .18	2.10 / .09	2.10 / .10	2.11 / .10	2.43 / .22	3.64 / .19	3.97 / .08			
			3.97 / .05	3.40 / .16	2.07 / .14	1.79 / .03	1.78 / .05	1.79 / .04	2.12 / .17	3.46 / .22	3.96 / .08			
-.003	-.005	-.008	3.96 / .06	3.21 / .18	2.05 / .11	1.79 / .04	1.77 / .05	1.80 / .04	2.12 / .15	3.28 / .23	3.95 / .08	-.012	-.007	-.005
			3.96 / .07	3.06 / .24	2.11 / .09	1.93 / .04	1.92 / .06	1.96 / .05	2.23 / .14	3.15 / .28	3.94 / .09			
			3.96 / .07	3.14 / .25	2.54 / .15	2.34 / .08	2.32 / .11	2.36 / .11	2.61 / .19	3.19 / .27	3.94 / .10			
			3.98 / .06	3.80 / .14	3.63 / .25	3.59 / .33	3.59 / .34	3.59 / .35	3.64 / .39	3.82 / .28	3.97 / .12			
			3.99 / .05	3.98 / .11	3.92 / .34	3.87 / .56	3.87 / .62	3.87 / .65	3.87 / .65	3.92 / .43	3.98 / .17			
							-.011							
							-.002							
							.0006							

TABLE IV.11 GRAVSAT - GEOSTATIONARY (2 1/2° x 2 1/2° Blocks)

A POSTERIORI DENSITY SIGMAS
10 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (1M,2.5M,5M)
HIGH SATELLITE STATE, σ [HCL]= (10M,25M,50M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							.00004							
							-.002							
							-.009							
			4.00 / .03	3.98 / .08	3.94 / .18	3.93 / .23	3.93 / .24	3.93 / .25	3.93 / .22	3.98 / .11	3.99 / .05			
			3.99 / .03	3.91 / .08	3.61 / .20	3.60 / .20	3.60 / .20	3.60 / .21	3.62 / .21	3.89 / .11	3.99 / .05			
			3.98 / .03	3.67 / .09	2.48 / .21	2.20 / .07	2.20 / .07	2.22 / .08	2.50 / .23	3.70 / .15	3.98 / .05			
			3.98 / .03	3.52 / .11	2.16 / .13	1.88 / .03	1.87 / .04	1.88 / .03	2.19 / .18	3.57 / .15	3.97 / .05			
-.003	-.004	-.007	3.97 / .04	3.31 / .14	2.13 / .11	1.87 / .04	1.86 / .04	1.88 / .04	2.20 / .15	3.35 / .17	3.96 / .06	-.011	-.007	-.005
			3.96 / .06	3.19 / .20	2.22 / .10	2.04 / .03	2.04 / .05	2.08 / .04	2.34 / .14	3.24 / .24	3.95 / .08			
			3.96 / .07	3.24 / .24	2.62 / .17	2.45 / .07	2.43 / .08	2.47 / .07	2.72 / .21	3.25 / .25	3.95 / .08			
			3.98 / .05	3.81 / .13	3.64 / .18	3.61 / .19	3.61 / .20	3.61 / .21	3.66 / .22	3.83 / .15	3.97 / .07			
			3.99 / .03	3.98 / .06	3.95 / .14	3.93 / .22	3.93 / .25	3.93 / .26	3.93 / .24	3.96 / .15	3.99 / .06			
							-.010							
							-.002							
							-.0001							

TABLE IV.III GRAVSAT - GEOSTATIONARY (2 1/2° x 2 1/2° Blocks)

A POSTERIORI DENSITY SIGMAS
10 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (2M,5M,10M)
HIGH SATELLITE STATE, σ [HCL]= (20M,50M,100M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							.003							
							.005							
							.011							
			4.00 / .004	4.00 / .010	3.99 / .02	3.99 / .02	3.99 / .02	3.99 / .02	3.99 / .02	3.99 / .01	4.00 / .006			
			4.00 / .006	3.96 / .02	3.78 / .06	3.77 / .06	3.76 / .06	3.77 / .06	3.79 / .07	3.96 / .03	4.00 / .008			
			4.00 / .004	3.92 / .02	3.36 / .06	3.25 / .04	3.25 / .04	3.25 / .04	3.38 / .07	3.91 / .03	4.00 / .007			
			3.99 / .010	3.72 / .04	2.55 / .07	2.47 / .05	2.47 / .05	2.48 / .06	2.59 / .08	3.72 / .05	3.99 / .02			
-.001	-.002	-.005	3.98 / .01	3.47 / .07	2.46 / .06	2.30 / .03	2.29 / .03	2.30 / .03	2.50 / .07	3.50 / .09	3.98 / .02	-.007	-.004	-.002
			3.99 / .009	3.58 / .05	2.67 / .07	2.55 / .05	2.55 / .05	2.56 / .06	2.69 / .09	3.63 / .05	3.99 / .01			
			3.99 / .01	3.71 / .06	3.47 / .05	3.36 / .04	3.35 / .04	3.36 / .04	3.48 / .06	3.72 / .07	3.99 / .02			
			3.99 / .01	3.90 / .04	3.80 / .06	3.77 / .06	3.77 / .06	3.77 / .06	3.82 / .06	3.92 / .04	3.99 / .01			
			4.00 / .006	3.99 / .01	3.99 / .02	3.99 / .02	3.99 / .02	3.99 / .02	3.99 / .02	3.99 / .01	4.00 / .008			
							.011							
							.005							
							.003							

TABLE IV.IV GRAVSAT - GEOSTATIONARY (2 1/2° x 2 1/2° Blocks)

A POSTERIORI DENSITY SIGMAS
10 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (40M,100M,200M)
HIGH SATELLITE STATE, σ [HCL]= (400M,1000M,2000M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							.012							
							.026							
							.019							
			7.99 .26	7.91 .88	7.62 2.34	7.66 2.75	7.67 2.85	7.69 2.96	7.66 2.78	7.88 1.30	7.97 .55			
			7.97 .14	7.69 .41	6.62 .94	6.56 .87	6.54 .87	6.57 .92	6.67 1.01	7.64 .63	7.95 .37			
			7.93 .12	6.70 .44	3.98 .52	3.17 .54	3.15 .56	3.18 .57	4.04 .76	6.85 .58	7.91 .29			
			7.91 .14	5.96 .51	3.01 .35	2.39 .16	2.36 .21	2.41 .22	3.14 .35	6.18 .72	7.89 .26			
-.005	-.008	-.012	7.90 .15	5.39 .48	2.89 .21	2.40 .13	2.36 .14	2.43 .15	3.04 .29	5.63 .63	7.86 .24	-.019	-.013	-.009
			7.89 .18	5.16 .57	3.06 .15	2.67 .22	2.65 .20	2.73 .20	3.26 .40	5.45 .70	7.85 .25			
			7.89 .17	5.49 .59	4.11 .41	3.69 .51	3.65 .58	3.74 .60	4.31 .69	5.64 .75	7.85 .26			
			7.94 .14	7.33 .32	6.75 .70	6.65 .93	6.62 .86	6.63 .87	6.83 1.12	7.41 1.02	7.92 .47			
			7.98 .17	7.94 .46	7.71 1.70	7.69 2.73	7.70 2.85	7.71 2.92	7.75 3.00	7.74 2.10	7.95 .81			
							.024							
							.030							
							.016							

TABLE IV.V GRAVSAT - GEOSTATIONARY (2 1/2° x 2 1/2° Blocks)

A POSTERIORI DENSITY SIGMAS
10 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 8 MGAL
LOW SATELLITE STATE, σ [HCL]= (1M,2.5M,5M)
HIGH SATELLITE STATE, σ [HCL]= (10M,25M,50M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 8 MGAL

							.004							
							.008							
							.016							
			8.00 .02	7.98 .04	7.95 .07	7.94 .08	7.94 .09	7.94 .09	7.95 .08	7.97 .05	7.99 .03			
			7.99 .02	7.85 .09	7.05 .24	7.00 .26	6.99 .25	7.01 .26	7.10 .26	7.83 .11	7.99 .03			
			7.99 .02	7.68 .07	5.73 .17	5.40 .10	5.40 .09	5.42 .10	5.79 .23	7.68 .08	7.98 .03			
			7.97 .03	6.94 .13	4.15 .15	3.79 .09	3.78 .10	3.81 .10	4.25 .19	7.01 .17	7.95 .05			
-.004	-.006	-.012	7.95 .04	6.20 .20	4.09 .10	3.73 .06	3.71 .06	3.75 .07	4.18 .14	6.33 .26	7.93 .07	-.018	-.010	-.007
			7.96 .03	6.42 .14	4.39 .15	3.98 .09	3.94 .10	4.00 .11	4.45 .20	6.58 .18	7.95 .05			
			7.97 .04	6.88 .19	6.01 .13	5.77 .10	5.72 .10	5.81 .11	6.09 .19	6.91 .25	7.95 .06			
			7.97 .04	7.58 .16	7.16 .24	7.04 .25	7.02 .25	7.05 .25	7.21 .24	7.65 .17	7.96 .06			
			7.99 .02	7.97 .05	7.95 .08	7.94 .08	7.94 .09	7.94 .09	7.95 .08	7.97 .06	7.99 .03			
							.017							
							.008							
							.004							

TABLE IV.VI GRAVSAT - GEOSTATIONARY (2 1/2° x 2 1/2° Blocks)

A POSTERIORI DENSITY SIGMAS
10 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 8 MGAL
LOW SATELLITE STATE, σ [HCL]= (40M,100M,200M)
HIGH SATELLITE STATE, σ [HCL]= (400M,1000M,2000M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 8 MGAL

							.002							
							.005							
							.0009							
			3.99 / .03	3.97 / .09	3.91 / .21	3.90 / .26	3.90 / .28	3.90 / .28	3.90 / .25	3.96 / .12	3.99 / .05			
			3.99 / .03	3.77 / .11	2.72 / .29	2.67 / .22	2.65 / .21	2.67 / .22	2.75 / .29	3.75 / .15	3.98 / .06			
			3.98 / .03	3.45 / .11	1.82 / .18	1.30 / .04	1.28 / .05	1.30 / .05	1.82 / .22	3.54 / .15	3.97 / .05			
			3.95 / .04	2.74 / .14	1.32 / .12	.89 / .03	.87 / .04	.90 / .04	1.35 / .15	2.84 / .19	3.94 / .07			
-.003	-.003	-.005	3.94 / .06	2.24 / .19	1.13 / .10	.90 / .03	.89 / .04	.92 / .04	1.21 / .13	2.34 / .26	3.92 / .07	.008	-.006	-.005
			3.95 / .07	2.24 / .19	1.26 / .09	1.03 / .03	1.01 / .04	1.07 / .04	1.41 / .12	2.36 / .25	3.93 / .09			
			3.95 / .08	2.65 / .27	1.81 / .11	1.66 / .04	1.65 / .07	1.72 / .05	2.04 / .17	2.72 / .30	3.93 / .10			
			3.96 / .07	3.42 / .23	2.89 / .27	2.79 / .21	2.78 / .21	2.80 / .21	2.93 / .28	3.51 / .22	3.95 / .09			
			3.99 / .05	3.96 / .09	3.92 / .18	3.90 / .25	3.90 / .28	3.90 / .39	3.90 / .27	3.94 / .17	3.99 / .07			
							-.001							
							.003							
							.002							

TABLE IV.VII GRAVSAT - GEOSTATIONARY (2 1/2° x 2 1/2° Blocks)

A POSTERIORI DENSITY SIGMAS
10 SHORT DATA ARCS (DATA NOISE = .01 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (2M,5M,10M)
HIGH SATELLITE STATE, σ [HCL]= (20M,50M,100M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							.002							
							.004							
							.002							
			3.99 / .04	3.96 / .09	3.90 / .21	3.89 / .26	3.89 / .28	3.88 / .28	3.89 / .26	3.95 / .13	3.99 / .06			
			3.98 / .04	3.68 / .12	2.25 / .33	2.02 / .21	2.01 / .21	2.03 / .22	2.29 / .33	3.68 / .17	3.98 / .06			
			3.98 / .03	3.30 / .11	1.56 / .16	1.00 / .04	9.77 / .05	9.97 / .05	1.61 / .21	3.41 / .15	3.97 / .06			
			3.96 / .05	2.07 / .17	1.06 / .11	.74 / .03	.72 / .04	.75 / .04	1.09 / .14	2.19 / .24	3.94 / .08			
-.003	-.003	-.004	3.95 / .06	1.81 / .19	9.35 / .10	.743 / .03	.74 / .04	.76 / .04	1.00 / .13	1.89 / .26	3.93 / .09	-.007	-.006	-.004
			3.95 / .06	1.78 / .19	.993 / .09	.800 / .03	.79 / .04	.83 / .04	1.11 / .13	1.87 / .26	3.92 / .10			
			3.95 / .08	2.39 / .26	1.53 / .10	1.36 / .04	1.36 / .06	1.43 / .05	1.73 / .15	2.51 / +3.0	3.93 / .10			
			3.96 / .08	3.14 / .28	2.48 / .30	2.38 / .21	2.26 / .22	2.32 / .21	2.50 / .32	3.22 / .27	3.94 / .10			
			3.99 / .05	3.95 / .11	3.91 / .18	3.89 / .25	3.89 / .28	3.88 / .29	3.89 / .28	3.93 / .18	3.98 / .08			
							-.0001							
							-.003							
							.002							

TABLE IV.VIII GRAVSAT - GEOSTATIONARY (2 1/2° x 2 1/2° Blocks)

A POSTERIORI DENSITY SIGMAS
10 SHORT DATA ARCS (DATA NOISE = .005 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (2M,5M,10M)
HIGH SATELLITE STATE, σ [HCL]= (20M,50M,100M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

			-.002	-.002	-.002	-.006	-.02	-.006	-.008	0	0			
			-.002	.002	-.005	-.03	-.03	-.03	-.007	.01	0			
			-.002	.01	-.01	-.01	.30	-.01	-.01	.01	-.002			
			-.01	-.02	-.01	.18	-.72	.18	-.002	-.02	-.007			
			-.007	-.04	.11	-.34	1	-.34	.11	-.04	-.01			
			-.001	.002	-.03	.19	-.70	.18	-.02	.005	-.002			
			.003	.04	-.04	.02	.24	.03	-.04	.04	.003			
			.001	.008	-.009	-.03	-.03	-.03	-.01	.007	-.001			
			-.001	-.002	-.006	-.005	-.02	-.006	-.001	-.003	-.003			

TABLE IV.IX

GRAVSAT - GEOSTATIONARY (2 1/2° x 2 1/2° Blocks)

CORRELATIONS OF ADJUSTED BLOCKS WITH CENTER BLOCK
10 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)
HIGH SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS σ = 4 MGAL

SET V:

Solution Set V investigates the use of the "high-high-low" GRAVSAT/ GEOSTATIONARY configuration described in Section 6.1 for the recovery of high latitude 5° x 5° equal-area blocks. Tables V.I-V.VI repeat the a priori uncertainties of the epoch states and density blocks found in the studies of Tables II.I-II.VII and Tables III.I-III.VII. Making the appropriate corrections to the resultant a posteriori adjusted block errors by dividing by $\sqrt{2}$ to account for the difference in data passes (see Section 6.1), we find gravity fine structure recovery comparable to the 5° x 5° equal-angular blocks located near the equator.

SET VI:

Solution Set VI as described in Section 6.1 is an extension of the "low-low" results obtained in report [A] and is included here for purposes of comparison with the "high-low" configurations of Sets I and II. Tables VI.I-VI.VII are designed to be compared with Tables I.I-I.VII and II.I-II.VII. The "high-low" results are superior in every case, although the degree of superiority is dependent on the assumed orbital knowledge of the low satellite. For the most favorable case assuming virtually error-free epoch states, the "high-low" mission is superior by a factor of approximately 20, while for assumed GRAVSAT epoch state uncertainties of (1m, 2.5m, 5m) and (2m, 5m, 10m) in HCL coordinates, the factor is approximately 10 and 7, respectively. For assumed GRAVSAT epoch state vectors with uncertainties of (40m, 100m, 200m) the superiority factor is only approximately 1.5. However, this comparison is somewhat misleadingly optimistic with regard to the "low-low" concept due to the fact that the orbital accuracy assumed is much more difficult to obtain for "low-low" short data arcs (see report [A]) than for "high-low" data arcs. Table VI.VIII displays the correlations for the error analysis study shown in Table VI.I, and reveals much higher values in the north-south direction (especially for nearest and next nearest neighbors) than the corresponding values for the "high-low" configurations of Sets I and II.

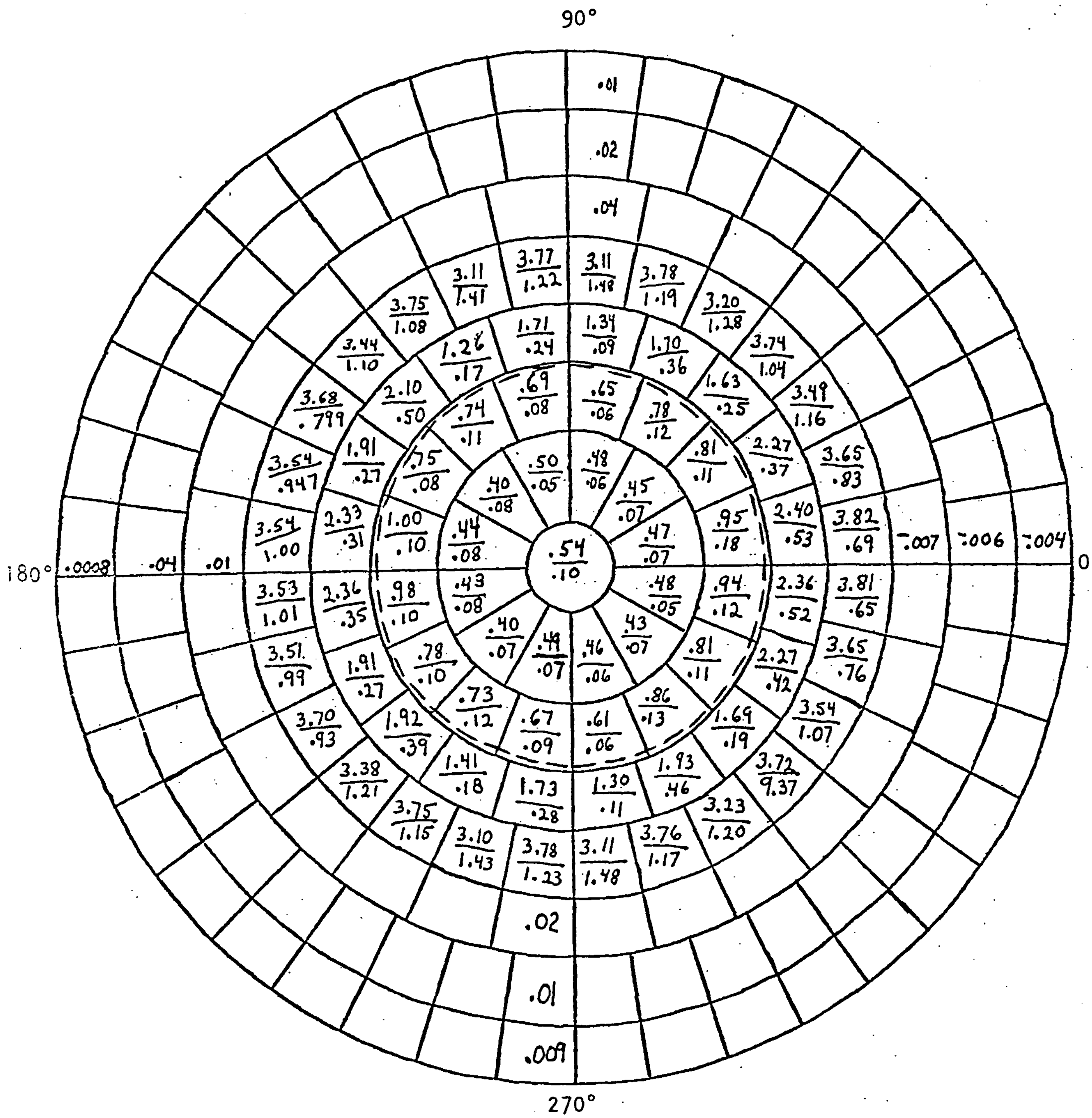


TABLE V.I GRAVSAT - 2 GEOSTATIONARY (POLAR 5° EQUAL-AREA BLOCKS)

A POSTERIORI DENSITY SIGMAS
12 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
77 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)
HIGH SATELLITE STATE, σ [HCL]= (.001M,.001M,.001M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
108 DENSITY BLOCKS, σ = 4 MGAL

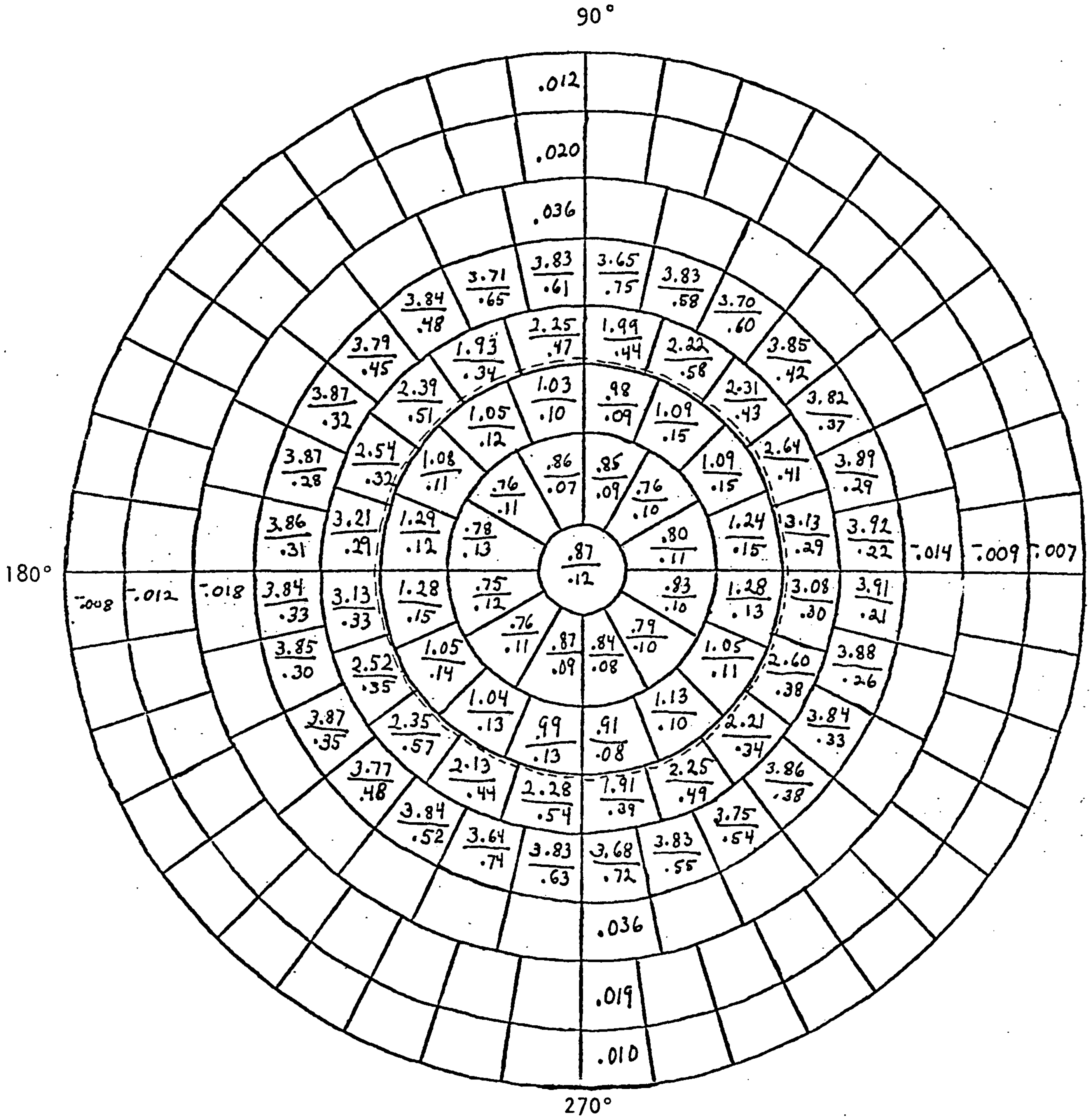


TABLE V.II GRAVSAT - 2 GEOSTATIONARY (POLAR 5° EQUAL-AREA BLOCKS)

A POSTERIORI DENSITY SIGMAS
12 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
77 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (1M,2.5M,5M)
HIGH SATELLITE STATE, σ [HCL]= (10M,25M,50M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
108 DENSITY BLOCKS, σ = 4 MGAL

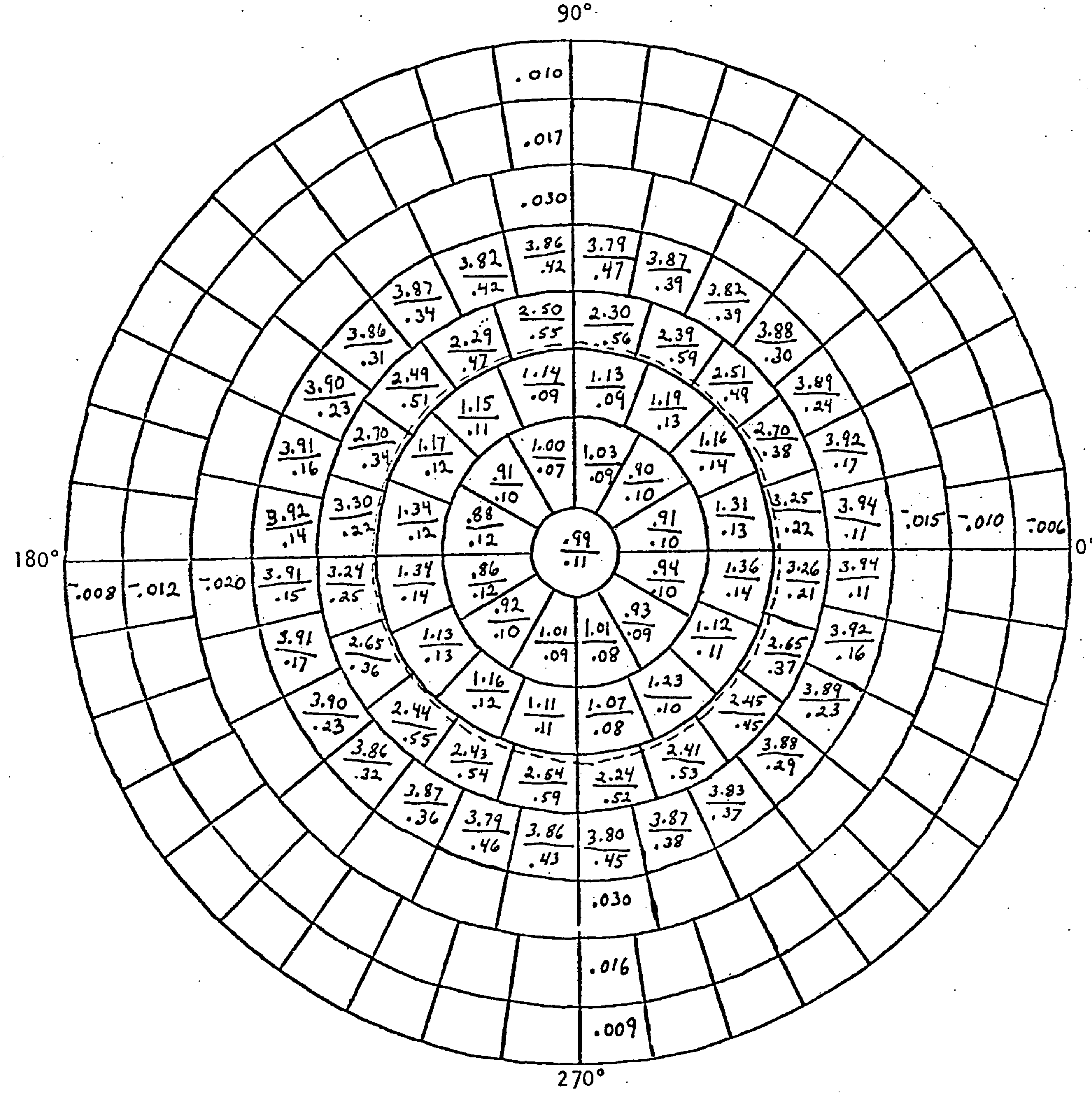


TABLE V.III GRAVSAT - 2 GEOSTATIONARY (POLAR 5° EQUAL-AREA BLOCKS)

A POSTERIORI DENSITY SIGMAS

12 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:

77 DENSITY BLOCKS, σ = 4 MGAL

LOW SATELLITE STATE, σ [HCL]= (2M,5M,10M)

HIGH SATELLITE STATE, σ [HCL]= (20M,50M,100M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:

108 DENSITY BLOCKS, σ = 4 MGAL

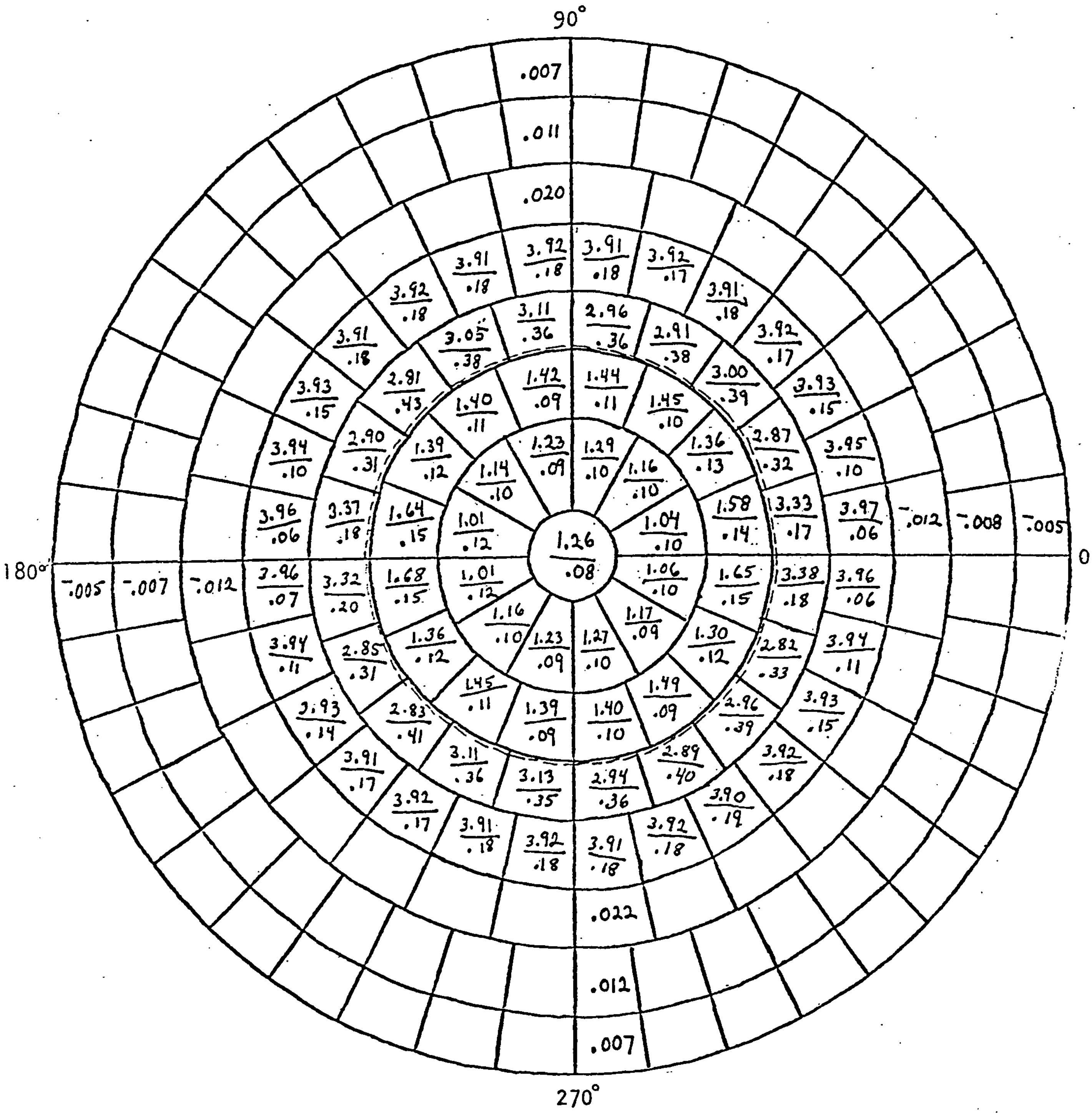


TABLE V.IV GRAVSAT - 2 GEOSTATIONARY (POLAR 5° EQUAL-AREA BLOCKS)

A POSTERIORI DENSITY SIGMAS

12 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:

77 DENSITY BLOCKS, σ = 4 MGAL

LOW SATELLITE STATE, σ [HCL]= (10M,25M,50M)

HIGH SATELLITE STATE, σ [HCL]= (100M,250M,500M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:

108 DENSITY BLOCKS, σ = 4 MGAL

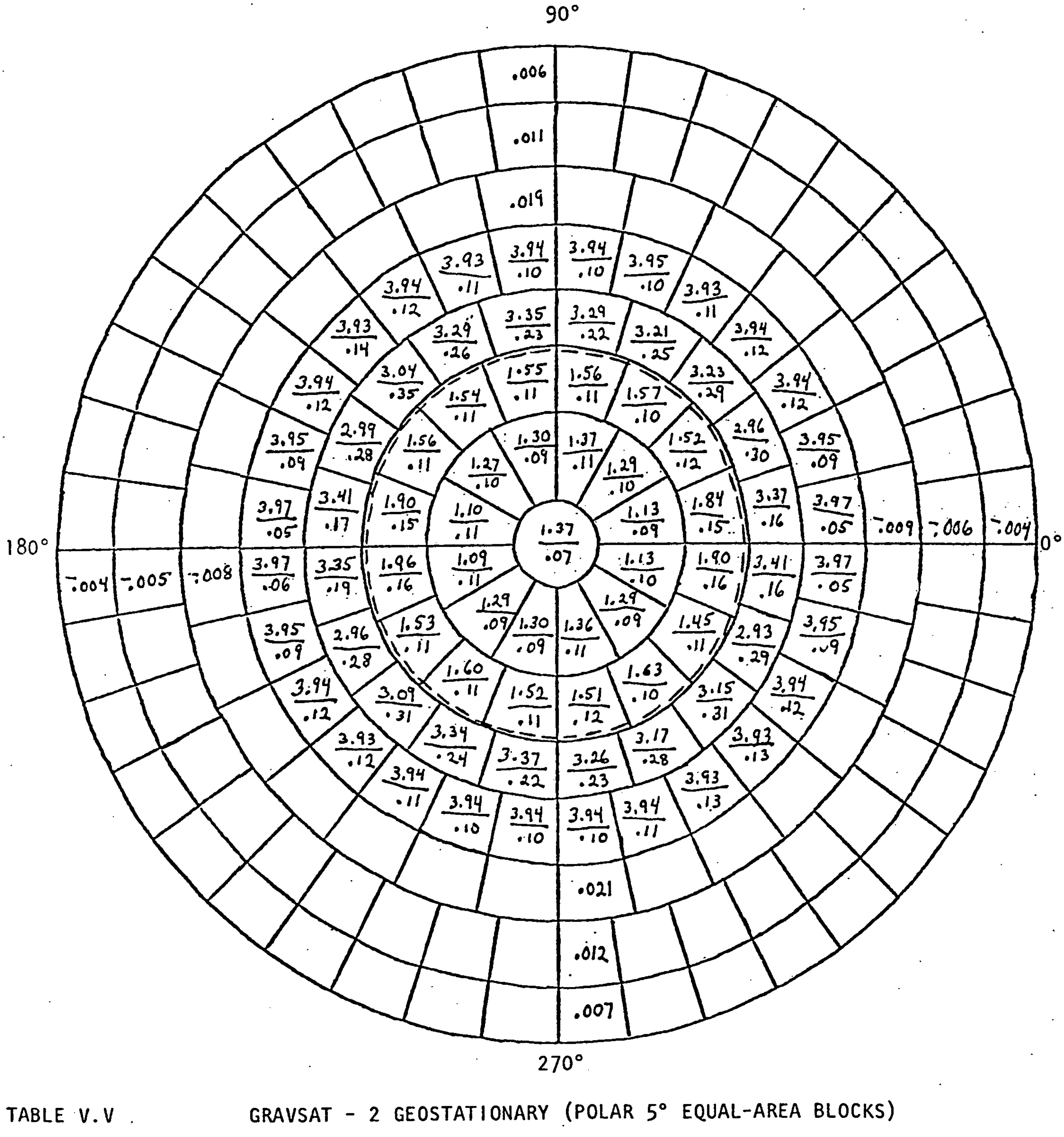


TABLE V.V GRAVSAT - 2 GEOSTATIONARY (POLAR 5° EQUAL-AREA BLOCKS)

A POSTERIORI DENSITY SIGMAS

12 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
77 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (20M,50M,100M)
HIGH SATELLITE STATE, σ [HCL]= (200M,500M,1000M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
108 DENSITY BLOCKS, σ = 4 MGAL

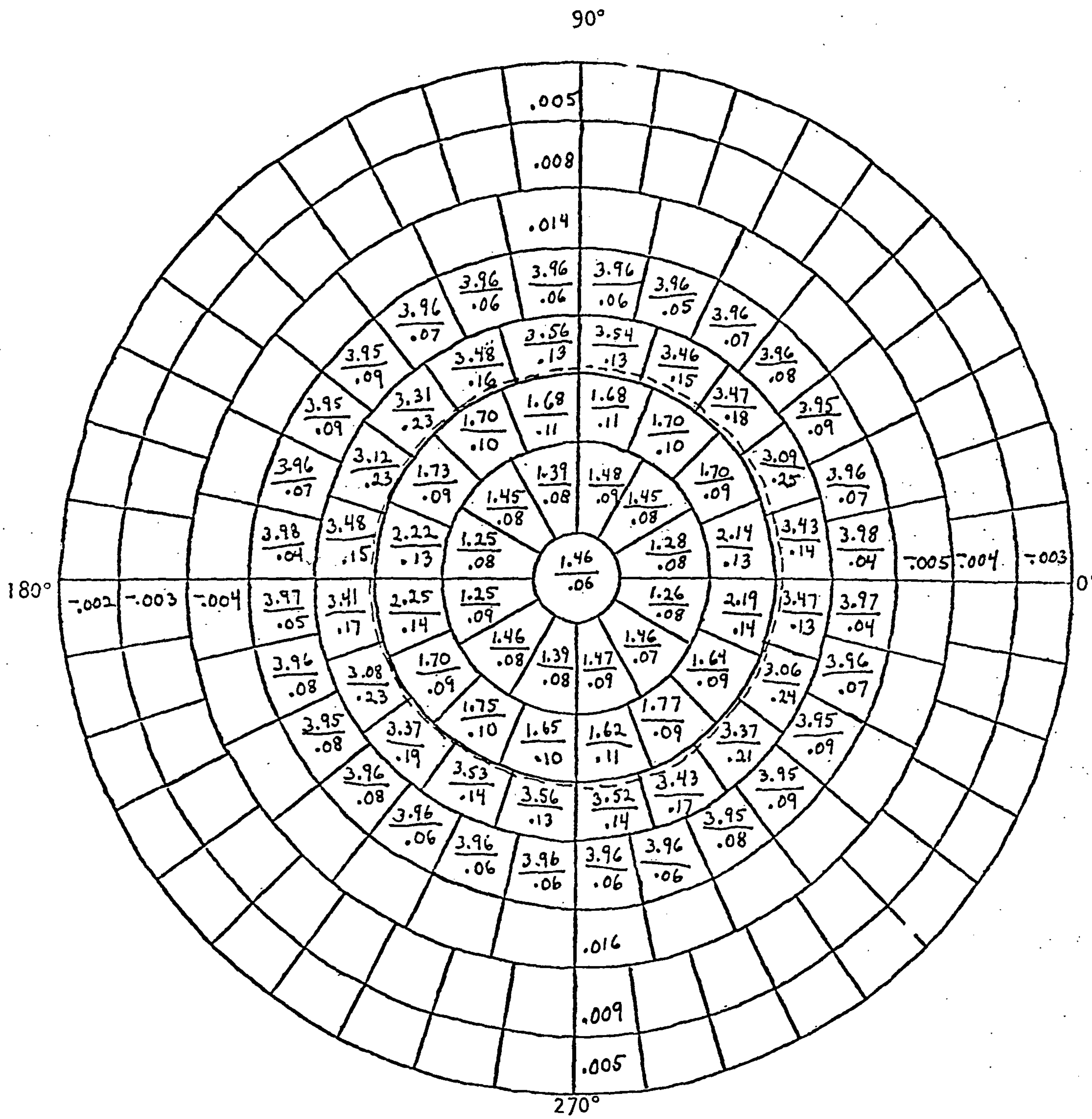


TABLE V.VI GRAVSAT - 2 GEOSTATIONARY (POLAR 5° EQUAL-AREA BLOCKS)

A POSTERIORI DENSITY SIGMAS
12 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
77 DENSITY BLOCKS, σ = 4 MGAL
LOW SATELLITE STATE, σ [HCL]= (40M,100M,200M)
HIGH SATELLITE STATE, σ [HCL]= (400M,1000M,2000M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
108 DENSITY BLOCKS, σ = 4 MGAL

							.019							
							.036							
							.08							
			3.98 .08	3.73 .30	3.42 .48	3.36 .40	3.34 .41	3.33 .41	3.40 .48	3.63 .36	3.97 .10			
			3.96 .06	3.04 .20	2.35 .24	2.26 .22	2.23 .22	2.22 .23	2.30 .24	2.79 .22	3.93 .07			
			3.94 .11	2.04 .30	1.82 .24	1.79 24	1.79 .24	1.79 .24	1.81 .25	1.99 .30	3.88 .13			
			3.91 .14	1.99 .26	1.80 .22	1.77 .21	1.76 .21	1.77 .21	1.81 .23	1.96 .27	3.80 .18			
-.005	-.008	-.01	3.91 .13	1.91 .27	1.68 .20	1.60 .17	1.60 .17	1.60 .17	1.67 .20	1.85 .29	3.81 .18	-.0097	-.007	-.005
			3.91 .10	1.66 .28	1.31 .18	1.19 .14	1.18 .14	1.18 .14	1.27 .18	1.54 .31	3.77 .17			
			3.94 .072	1.56 .28	.97 .21	.82 .17	.81 .17	.82 .17	.89 .19	1.26 .32	3.86 .11			
			3.97 .05	2.96 .21	1.62 .27	1.46 .24	1.49 .23	1.52 .24	1.52 .25	2.22 .30	3.93 .081			
			3.99 .02	3.85 .15	3.58 .28	3.50 .24	3.51 .24	3.53 .23	3.57 .27	3.74 .23	3.99 .04			
							-.04							
							-.006							
							-.0008							

TABLE VI.I

GRAVSAT - GRAVSAT (LOW-LOW)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
SATELLITE STATES, σ [HCL]= (.001M,.001M,.001M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							.18							
							.35							
							.92							
			49.1 / 7.2	37.3 / 8.1	26.4 / 7.8	24.8 / 7.5	24.6 / 7.6	25.0 / 7.7	25.1 / 7.8	35.4 / 8.3	47.8 / 3.2			
			47.0 / 1.9	23.4 / 3.5	14.2 / 2.4	13.4 / 2.3	13.4 / 2.2	13.9 / 2.2	14.4 / 2.4	20.6 / 2.6	42.3 / 4.4			
			42.6 / 4.6	20.3 / 7.4	12.4 / 2.3	11.4 / 2.2	11.6 / 2.1	12.3 / 2.1	13.2 / 2.4	17.9 / 2.8	32.4 / 7.7			
			41.2 / 4.7	18.7 / 3.9	11.1 / 2.3	9.9 / 2.0	10.1 / 1.9	11.0 / 1.9	12.3 / 2.4	16.6 / 3.4	30.4 / 7.5			
-.05	-.06	-.09	41.8 / 4.3	17.5 / 4.3	9.6 / 2.2	8.3 / 1.9	8.5 / 1.8	9.3 / 1.8	10.8 / 2.3	15.3 / 3.7	30.3 / 7.0	-.13	-.08	-.06
			40.0 / 4.7	14.7 / 4.1	7.8 / 2.2	6.5 / 1.7	6.7 / 1.8	7.3 / 1.9	8.6 / 2.9	13.1 / 3.8	28.0 / 7.0			
			43.4 / 3.8	12.0 / 3.8	6.4 / 2.3	5.5 / 1.9	5.6 / 1.9	5.8 / 1.9	6.6 / 2.3	10.9 / 3.8	29.8 / 7.7			
			46.8 / 2.6	16.3 / 3.6	7.5 / 2.4	7.1 / 2.1	7.1 / 2.0	7.1 / 2.1	7.2 / 2.4	12.1 / 3.5	42.0 / 4.6			
			49.3 / 1.3	39.8 / 6.2	21.7 / 7.8	20.1 / 8.3	20.1 / 8.4	20.1 / 8.1	20.0 / 7.8	34.0 / 8.6	47.7 / 3.1			
							.23							
							.21							
							.09							

TABLE VI.11

GRAVSAT - GRAVSAT (LOW-LOW)

A POSTERIORI DENSITY SIGMAS

21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 50 MGAL
SATELLITE STATES, σ [HCL]= (.001M,.001M,.001M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 50 MGAL

							.015							
							.026							
							.045							
			3.99 / .04	3.82 / .20	3.54 / .38	3.49 / .35	3.47 / .35	3.47 / .36	3.51 / .38	3.74 / .25	3.98 / .06			
			3.97 / .05	3.22 / .16	2.47 / .24	2.37 / .21	2.34 / .22	2.32 / .23	2.42 / .24	2.94 / .18	3.94 / .07			
			3.94 / .09	2.09 / .27	1.89 / .21	1.85 / .22	1.85 / .22	1.85 / .22	1.88 / .22	2.03 / .28	3.89 / .11			
			3.93 / .09	2.09 / .20	1.87 / .18	1.84 / .18	1.84 / .18	1.84 / .18	1.87 / .18	2.04 / .22	3.86 / .13			
-.005	-.008	-.011	3.94 / .07	2.12 / .16	1.81 / .15	1.77 / .14	1.76 / .13	1.76 / .13	1.80 / .16	2.01 / .18	3.85 / .10	-.009	-.007	-.005
			3.93 / .07	2.02 / .18	1.63 / .14	1.57 / .12	1.56 / .11	1.56 / .12	1.60 / .15	1.90 / .21	3.85 / .11			
			3.94 / .07	2.01 / .23	1.53 / .19	1.47 / .16	1.46 / .16	1.46 / .16	1.50 / .19	1.83 / .25	3.88 / .10			
			3.97 / .05	3.27 / .14	2.15 / .26	2.07 / .23	2.09 / .22	2.11 / .22	2.15 / .26	2.78 / .20	3.94 / .07			
			3.99 / .02	3.86 / .14	3.63 / .26	3.57 / .22	3.58 / .22	3.59 / .21	3.62 / .25	3.76 / .20	3.99 / .04			
							-.034							
							-.005							
							-.0009							

TABLE VI.III

GRAVSAT - GRAVSAT (LOW-LOW)

A POSTERIORI DENSITY SIGMAS

21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
SATELLITE STATES, σ [HCL]= (1M,2.5M,5M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							.008							
							.013							
							.011							
			3.99 / .03	3.85 / .15	3.63 / .28	3.58 / .27	3.56 / .27	3.56 / .28	3.61 / .29	3.79 / .19	3.99 / .04			
			3.97 / .05	3.24 / .16	2.50 / .23	2.40 / .21	2.38 / .22	2.36 / .22	2.45 / .23	2.96 / .18	3.95 / .07			
			3.94 / .08	2.18 / .24	1.94 / .19	1.89 / .19	1.88 / .19	1.89 / .19	1.92 / .19	2.08 / .26	3.89 / .10			
			3.94 / .07	2.15 / .17	1.94 / .13	1.90 / .14	1.89 / .14	1.90 / .14	1.93 / .14	2.09 / .19	3.88 / .10			
-.004	-.006	-.010	3.95 / .04	2.16 / .13	1.90 / .09	1.86 / .09	1.86 / .09	1.86 / .09	1.89 / .09	2.07 / .15	3.86 / .08	-.009	-.006	-.004
			3.94 / .06	2.10 / .16	1.79 / .09	1.73 / .08	1.73 / .08	1.73 / .08	1.76 / .10	2.00 / .19	3.87 / .09			
			3.95 / .07	2.13 / .22	1.75 / .17	1.69 / .15	1.68 / .14	1.68 / .15	1.71 / .16	1.97 / .25	3.88 / .09			
			3.97 / .05	3.31 / .14	2.33 / .24	2.25 / .22	2.27 / .22	2.29 / .21	2.31 / .24	2.85 / .20	3.94 / .07			
			4.00 / .02	3.87 / .13	3.65 / .24	3.59 / .21	3.60 / .20	3.66 / .20	3.64 / .23	3.77 / .19	3.99 / .04			
							.030							
							.006							
							.001							

TABLE VI.IV GRAVSAT - GRAVSAT (LOW-LOW)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
SATELLITE STATES, σ [HCL]= (2M,5M,10M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							-.003							
							-.009							
							-.036							
			4.00 .009	3.92 .08	3.78 .16	3.75 .14	3.74 .14	3.74 .15	3.77 .17	3.87 .11	4.00 .02			
			3.97 .04	3.31 .14	2.57 .20	2.46 .19	2.43 .19	2.41 .20	2.50 .21	3.01 .16	3.95 .05			
			3.96 .04	2.53 .14	2.13 .14	2.08 .13	2.07 .12	2.08 .13	2.11 .14	2.35 .17	3.91 .06			
			3.96 .04	2.30 .11	2.09 .09	2.04 .08	2.03 .07	2.03 .07	2.06 .09	2.21 .13	3.91 .06			
-.0009	-.002	-.003	3.95 .03	2.20 .12	1.99 .07	1.97 .06	1.97 .06	1.97 .06	1.98 .06	2.12 .13	3.87 .06	-.002	-.001	-.0008
			3.96 .04	2.29 .12	2.03 .10	1.99 .08	1.98 .08	1.98 .08	2.02 .10	2.18 .14	3.90 .06			
			3.96 .05	2.53 .15	2.08 .16	2.02 .14	2.02 .14	2.02 .14	2.05 .16	2.30 .18	3.90 .07			
			3.98 .04	3.39 .13	2.50 .22	2.40 .21	2.42 .21	2.44 .20	2.48 .22	2.93 .19	3.95 .06			
			4.00 .01	3.91 .08	3.73 .19	3.70 .17	3.70 .17	3.71 .16	3.73 .19	3.84 .13	3.99 .02			
							-.029							
							-.007							
							-.002							

TABLE VI.V GRAVSAT - GRAVSAT (LOW-LOW)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
SATELLITE STATES, σ [HCL]= (10M,25M,50M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							-.004							
							-.010							
							-.038							
			4.00 / .005	3.94 / .053	3.83 / .11	3.81 / .11	3.80 / .11	3.80 / .11	3.82 / .12	3.91 / .07	4.00 / .010			
			3.98 / .03	3.34 / .12	2.61 / .17	2.49 / .16	2.47 / .17	2.45 / .17	2.54 / .17	3.04 / .14	3.95 / .04			
			3.97 / .03	2.62 / .11	2.22 / .09	2.18 / .09	2.18 / .09	2.18 / .09	2.21 / .10	2.46 / .13	3.92 / .05			
			3.96 / .03	2.33 / .10	2.13 / .07	2.09 / .06	2.08 / .06	2.08 / .06	2.11 / .07	2.25 / .12	3.91 / .05			
-.0006	-.001	-.002	3.96 / .03	2.20 / .12	2.00 / .07	1.98 / .06	1.97 / .06	1.97 / .06	1.99 / .07	2.12 / .13	3.87 / .06	-.0009	-.0008	-.0005
			3.96 / .03	2.34 / .10	2.09 / .07	2.06 / .07	2.06 / .06	2.06 / .07	2.08 / .07	2.23 / .12	3.91 / .05			
			3.97 / .03	2.63 / .12	2.20 / .11	2.16 / .11	2.15 / .10	2.16 / .11	2.17 / .11	2.42 / .13	3.91 / .05			
			3.98 / .03	3.41 / .11	2.55 / .19	2.45 / .18	2.46 / .18	2.48 / .18	2.53 / .19	2.96 / .16	3.95 / .05			
			4.00 / .006	3.94 / .06	3.80 / .13	3.78 / .13	3.78 / .13	3.78 / .13	3.79 / .14	3.89 / .09	4.00 / .01			
							-.031							
							-.007							
							-.002							

TABLE VI.VI

GRAVSAT - GRAVSAT (LOW-LOW)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
SATELLITE STATES, σ [HCL]= (20M,50M,100M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

							-.003							
							-.010							
							-.037							
			4.00 / .003	3.95 / .04	3.88 / .07	3.86 / .07	3.85 / .07	3.85 / .07	3.87 / .07	3.93 / .05	4.00 / .007			
			3.98 / .03	3.36 / .10	2.66 / .14	2.54 / .13	2.51 / .13	2.50 / .14	2.59 / .14	3.06 / .13	3.96 / .04			
			3.97 / .02	2.66 / .10	2.30 / .05	2.27 / .05	2.27 / .05	2.27 / .05	2.29 / .05	2.52 / .11	3.92 / .04			
			3.96 / .02	2.35 / .09	2.16 / .06	2.12 / .05	2.11 / .05	2.11 / .05	2.14 / .06	2.27 / .11	3.92 / .04			
-.0005	-.001	-.002	3.96 / .03	2.20 / .13	2.00 / .07	1.98 / .06	1.98 / .06	1.98 / .06	1.99 / .07	2.12 / .13	3.87 / .06	-.0007	-.0007	-.0004
			3.97 / .02	2.36 / .09	2.14 / .06	2.11 / .05	2.11 / .05	2.11 / .05	2.13 / .05	2.26 / .11	3.91 / .04			
			3.97 / .02	2.68 / .11	2.30 / .05	2.26 / .06	2.26 / .06	2.26 / .06	2.27 / .06	2.50 / .11	3.92 / .05			
			3.98 / .03	3.43 / .09	2.61 / .15	2.50 / .14	2.51 / .14	2.53 / .14	2.58 / .15	2.99 / .14	3.95 / .04			
			4.00 / .003	3.95 / .04	3.86 / .08	3.84 / .08	3.84 / .08	3.84 / .08	3.85 / .08	3.92 / .06	4.00 / .008			
							-.034							
							-.008							
							-.003							

TABLE VI.VII GRAVSAT - GRAVSAT (LOW-LOW)

A POSTERIORI DENSITY SIGMAS
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
SATELLITE STATES, σ [HCL]= (40M,100M,200M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

			-.002	-.01	.01	.13	.13	.15	0	-.009	-.003			
			-.001	.01	-.11	.09	.43	.09	-.10	.008	-.01			
			-.005	-.008	-.08	-.13	.85	-.13	-.06	-.02	-.01			
			-.01	-.03	-.03	-.26	.97	-.26	-.01	-.04	0			
			-.01	-.04	-.02	-.33	1	-.33	.02	-.04	-.003			
			-.01	-.04	.04	-.34	.96	-.35	.05	-.03	-.02			
			-.008	-.01	.05	-.22	.61	-.24	.06	-.004	.02			
			-.003	-.002	.023	.12	-.29	.10	.03	.005	.001			
			-.001	.003	.05	.006	-.23	-.02	.05	.002	.002			

TABLE VI.VIII

GRAVSAT - GRAVSAT (LOW-LOW)

CORRELATIONS OF ADJUSTED BLOCKS WITH CENTER BLOCK
21 SHORT DATA ARCS (DATA NOISE = .05 CM/SEC)

ADJUSTED PARAMETERS AND A PRIORI SIGMAS:
81 DENSITY BLOCKS, σ = 4 MGAL
SATELLITE STATES, σ [HCL]= (.001M,.001M,.001M)

UNADJUSTED PARAMETERS AND UNCERTAINTIES:
144 DENSITY BLOCKS, σ = 4 MGAL

7.0 SUMMARY

This study applied covariance error analysis techniques to investigate estimation strategies for the "high-low" mission for accurate local recovery of gravitational fine structure from multiple short data arcs, considering the effects of aliasing. The low drag-free GRAVSAT spacecraft was configured in a circular polar orbit at 250 km. altitude, while various configurations of the GEOPAUSE and geosynchronous spacecraft were utilized as high relay satellites. Sets of 5° x 5° equal-angular blocks near the equator and 5° x 5° equal-area blocks near the poles were used to represent the surface density block model of the high order geopotential used in the study.

Satellite-to-satellite relative range-rate data was used in the error analysis investigation, as range data was found to be much less sensitive to fine structure recovery. A data noise of .05 cm/sec and a data rate of one observation every 5 seconds were assumed for most of the error analysis studies. No ground tracking data was included and satellite state accuracy requirements of the estimation of the density blocks were investigated in the error analysis by a priori error sigmas for the epoch states. The sensitivity of the data type to small a priori errors in the low satellite epoch state velocities led to severe aliasing errors in the estimation of local sets of density blocks, forcing the GRAVSAT epoch state parameters to be included with the surface density parameters in the estimation process. The estimation of local sets of density blocks was much less sensitive to the epoch state errors of the high relay satellite, so that for reasonable orbital uncertainties (order of tens of meters) the high satellite epoch state parameters could be treated as unadjusted in the estimation process.

The error analysis for the recovery of local sets of density blocks using independent short data arcs demonstrated that the estimation strategy of simultaneously estimating a local set of density blocks covered by data and two surrounding "buffer layers" of density blocks not covered by data resulted in almost negligible aliasing errors on the estimation accuracy for blocks near the center of the recovered set due to unadjusted surface dnesity blocks. Estimates of blocks away from the center of the recovered set should be discarded. Error analysis studies using equal-area blocks at high latitudes (near the pole) gave results comparable to those obtained with density blocks near the equator. An extrapolation to 5° x 5° resolution from a numerical simulation performed for global density block recovery using long data arcs indicates that local recoveries, using estimation strategies to reduce aliasing errors, can produce comparable recovery accuracies.

Tables 7.1-7.3 present a comparison summary between the recovery accuracies for the "high-low" GRAVSAT/GEOPAUSE and GRAVSAT/GEOSTATIONARY configurations and the "low-low" configuration. The results presented in mgals are relative to gravity anomaly blocks. They are obtained by multiplying the results of Section 6.0 by 2π (see Section 2.0). The summary tables clearly display the superiority of the "high-low" mission over the "low-low". For the low satellite assumed accurate to 2 meters radial, 5 meters cross-track, and 10 meters along-track, and the geostationary satellite assumed accurate to 20 meters radial, 50 meters cross-track and 100 meters along-track, then a priori density block errors of 25 mgals can be reduced to 2 mgals. For these orbital errors multiplied by 20, the a priori density block errors can be reduced to 8 mgals.

These recovery accuracies are comparable to those obtained by Argentiero [1] in simulating gravity anomaly recovery using gradiometer data. We conclude, contrary to the findings for the "low-low" SST mission, that knowledge of the fine structure of the earth's gravitational structure can be significantly improved at the 5° x 5° resolution using satellite-to-satellite tracking data from a "high-low" satellite configuration. However, reasonably accurate orbits must be available for the low GRAVSAT.

The study demonstrates that the "high-low" SST data with the low satellite at 250 km. altitude and a measurement noise of .05 cm/sec does not significantly improve the gravitational fine structure at the 2 1/2° x 2 1/2° resolution. A factor of ten improvement in the measurement noise (to .005 cm/sec) is needed to establish the feasibility of gravity fine structure recovery at 2 1/2° resolution from SST range-rate data.

TABLE 7.1 GRAVSAT-GEOSTATIONARY

	STATE σ's (meters)			GRAVITY σ's (mgals) A Priori = 25 mgals
	H	C	L	
LOW:	1	2.5	5	1.2
HIGH:	10	25	50	
LOW:	2	5	10	1.7
HIGH:	20	50	100	
LOW:	10	25	50	5.2
HIGH:	100	250	500	
LOW:	20	50	100	7.2
HIGH:	200	500	1000	
LOW:	40	100	200	8.5
HIGH:	400	1000	2000	

TABLE 7.2 GRAVSAT-GEOPAUSE

	STATE σ's (meters)			GRAVITY σ's (mgals) A Priori = 25 mgals
	H	C	L	
LOW:	2	5	10	1.88
HIGH:	.4	1	2	
LOW:	10	25	50	5.4
HIGH:	2	5	10	
LOW:	20	50	100	7.16
HIGH:	4	10	20	
LOW:	20	50	100	7.48
HIGH:	200	500	1000	
LOW:	40	100	200	8.67
HIGH:	400	1000	2000	

TABLE 7.3 GRAVSAT-("LOW-LOW")

	STATE σ's (meters)			GRAVITY σ's (mgals) A Priori = 25 mgals
	H	C	L	
LOW:	1	2.5	5	11
LOW:	2	5	10	11.7
LOW:	10	25	50	12.4
LOW:	20	50	100	12.5
LOW:	40	100	200	12.5

8.0 NEW TECHNOLOGY

The effort under this contract consisted of the application of covariance error analysis techniques for investigation of estimation strategies for the "high-low" SST mission for accurate local recovery of gravitational fine structure, considering the effects of aliasing errors. Frequent reviews and a final survey for new technology were performed. It is believed that the mathematical and programming techniques and algorithms developed do not represent "reportable items", or patentable items, within the meaning of the New Technology Clause. Our reveiws and final survey found no other items which could be considered reportable items under the New Technology Clause.

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APPENDIX A. GLOBAL LONG-ARC SOLUTIONS FOR THE "LOW-LOW" CONFIGURATION

The results of report [A] demonstrated that aliasing from unadjusted density blocks led to severe corruption of the estimates of adjusted blocks when long data arcs were used in a local recovery. To establish the relationship between accuracies obtainable from global recoveries (no aliasing from unadjusted density blocks) and a global solution obtained as a composite of short-arc local recoveries, an error analysis experiment was performed to simulate a global 5° x 5° equal-area block recovery via long-arcs.

The experiment was to scale the "low-low" satellite geometry and the rotation rate of the earth so as to obtain global error analysis solutions for smaller and smaller density block sizes and to observe the character of the global solutions as they approached the 5° x 5° block local solutions of report [A]. The satellite configuration consisted of two spacecraft in polar circular orbits separated by the length of the equatorial equal-area block and at altitudes equal to the length of the equatorial block. The rotation rate of the earth was set to

$$\omega_E = \frac{2\pi}{nP}$$

where n is the number of equal-area blocks on the equator and P is the satellite orbital period. The data rate was such that there were four measurements per pass per block at the equator, and a single data arc of duration

$$T_A = nP$$

was used which produced two passes at the equator per block. This experiment was performed for global sets of 90° (n=4), 45° (n=8), 30° (n=12), 20° (n=18), 15° (n=24) and 10° (n=36) equal-area blocks. The SST relative range-rate measurements were given .05 cm/sec standard deviation, the a priori sigmas of the density blocks were set at 1 mgal and 50 mgal, and the satellite epoch states were included as adjusted parameters in the simulated recovery.

Figures A.1.1 - A.1.3 display the a posteriori error sigmas for the most favorable case of 20° x 20° equal-area blocks for 1 mgal and 50 mgal a priori on the density blocks, respectively, with virtually perfectly known orbital epoch states. The smaller errors near the poles reflect the more dense data coverage. Figure A.1.3 displays the correlation pattern of the global set of 105 20° x 20° blocks with block number 44.

Figure A.1.4 displays the extrapolation of the sigmas for the equatorial blocks of all the solutions assuming perfectly known orbital states to a 5° x 5° global set for 1 mgal and 50 mgal a priori. The extrapolated value of .8 mgal for the case of 1 mgal a priori compares favorably with the value for the center 5° x 5° equal-angular block of Table II.3 of report [A] (.48 mgal) when corrected for the difference in data coverage between the two solutions, i.e.,

$$\text{Factor} = \sqrt{\frac{\text{Passes per block report [A]}}{\text{Passes per block Appendix A}} \cdot \frac{\text{Data points per block report [A]}}{\text{Data points per block Appendix A}}}$$

$$= \sqrt{\frac{4}{2} \cdot \frac{15}{4}}$$

$$\simeq 2.74,$$

so that the global recovery error becomes

$$.8 \text{ mgal/FACTOR} \simeq .30 \text{ mgal.}$$

This result indicates the equivalence of local short-arc recoveries for gravitational fine structure using the "low-low" configuration, when strategies are employed to reduce aliasing, with global recoveries using long data arcs.

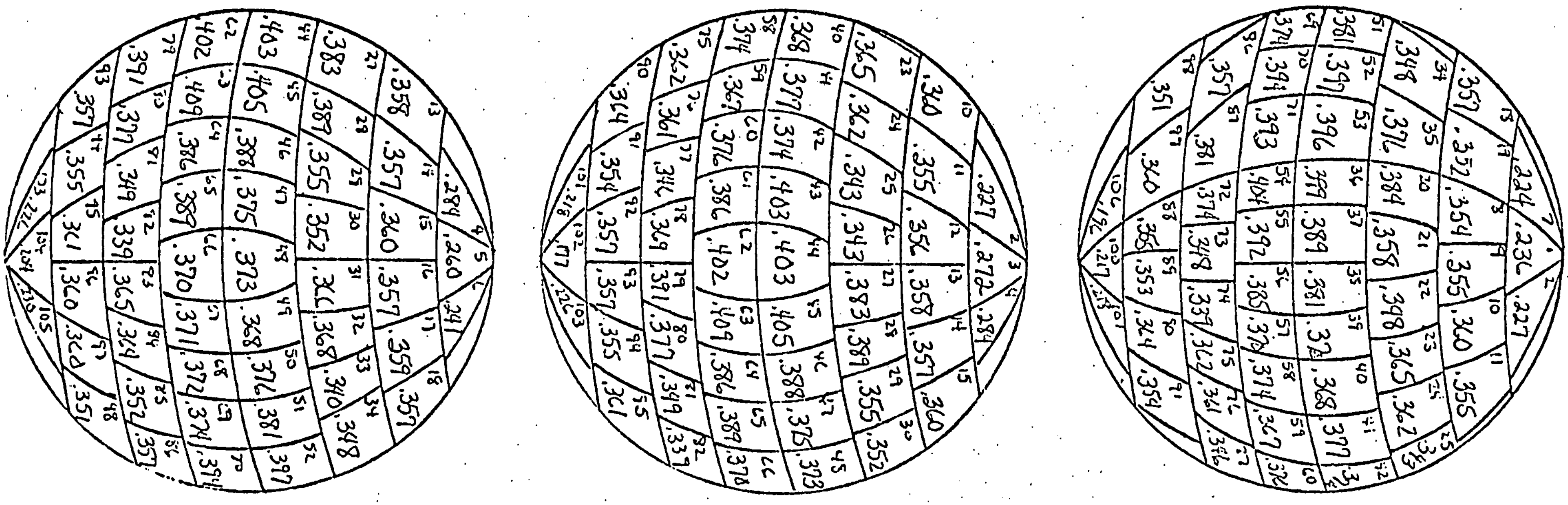

FIGURE A.1.1 Low-Low Long Arc A Posteriori Density Sigmas:One Long Arc (Data Noise = .05 Cm/Sec).

Adjusted Parameters and A Priori Sigmas:

105 20° Equal-Area Density Blocks, σ = 1 MGAL
Satellite States, σ[HCL]= (.001M,.001M,.001M).

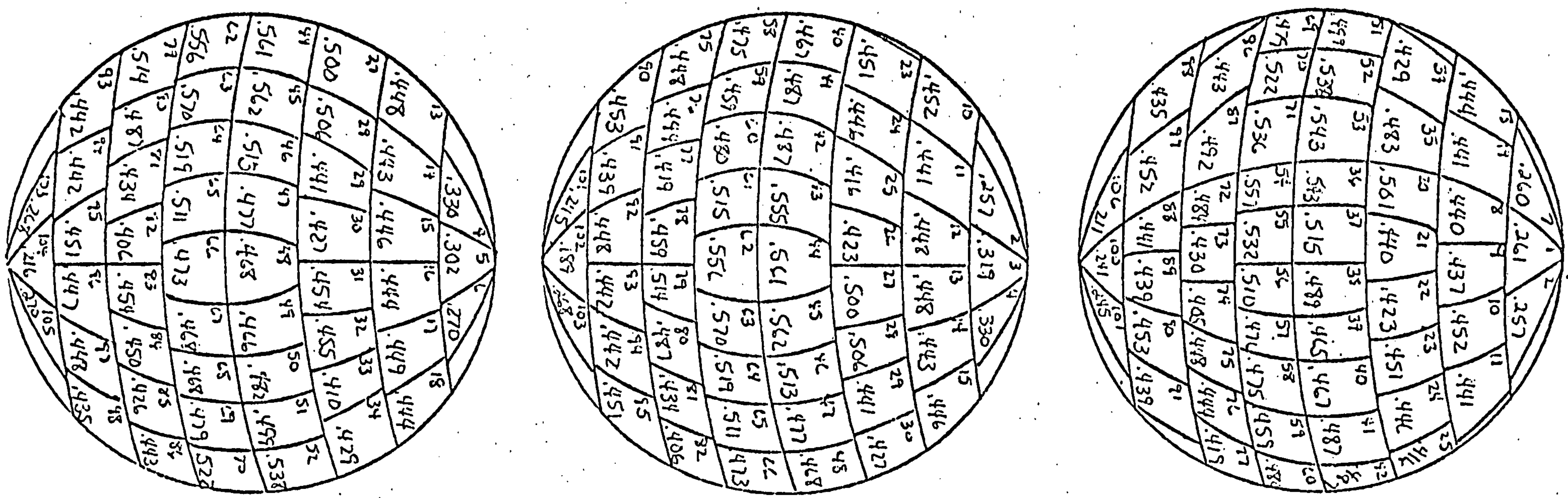

FIGURE A.I.2 Low-Low Long Arc A Posteriori Density Sigmas: One Long Arc (Data Noise = .05 Cm.Sec).

Adjusted Parameters and A Priori Sigmas:

105 20° Equal-Area Density Blocks, σ = 50 MGAL
Satellite States, σ[HCL]= (.001M,.001M,.001M).

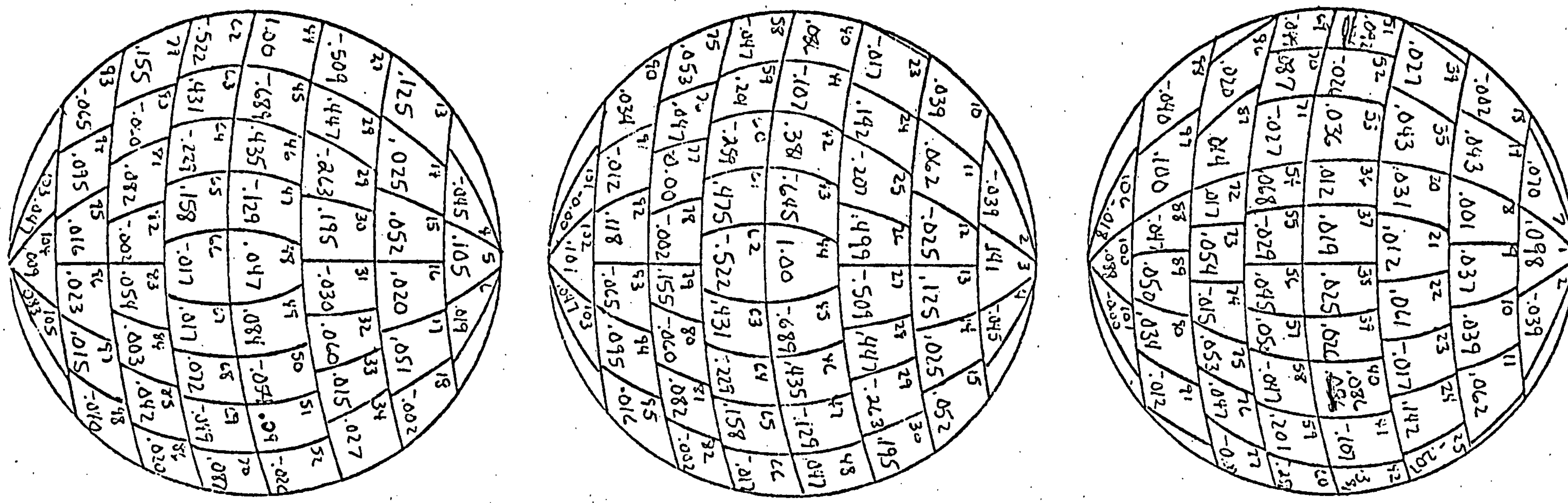

FIGURE A.I.3 Low-Low Long Arc Correlations with Density Block 44: One Long Arc (Data Noise = .05 Cm/Sec).

Adjusted Parameters and A Priori Sigmas:

105 20° Equal-Area Blocks, σ = 50 MGAL
Satellite States, σ[HCL] = (.001M,.001M,.001M).

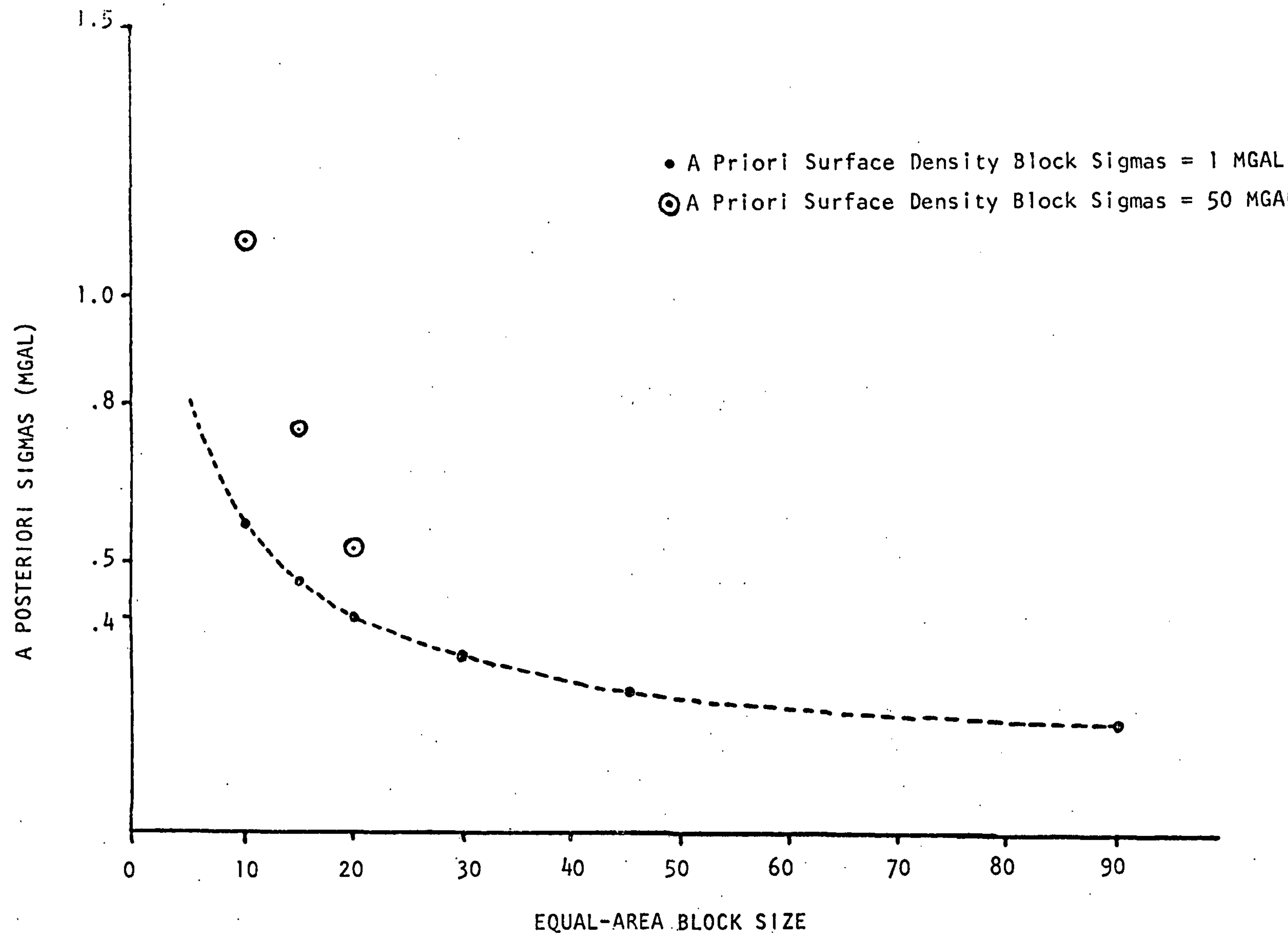

FIGURE A.1.4 Recovery Accuracy For Long Arc "Low-Low" Configuration With Global Coverage Over Equal-Area Density Blocks.